Basic concepts and quantities in electrical engineering. Ohm's law and power. Active and passive elements in DC networks. Kirchhoff's voltage and current laws in DC networks. Methods for analysis of linear electrical networks.

1.1. Basic concepts and quantities in electrical engineering

1.1.1. Electric charges

There are two forms of matter – substance and field. Substance consists of elementary particles which could be electrically neutral (neutrons) or electrically charged.

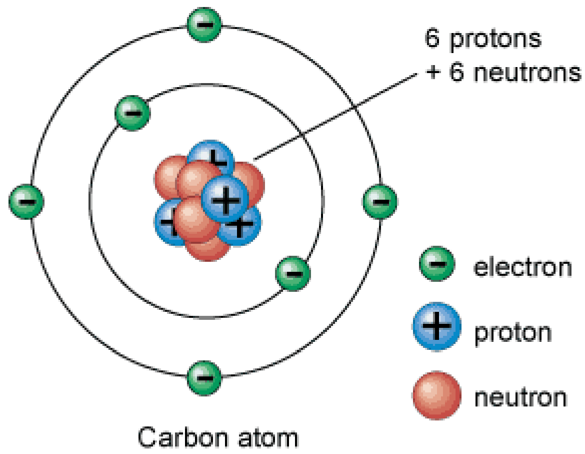


Fig. 1.1. Carbon atom

The electrically charged particles could have a positive (protons) or negative (electrons) charge. The unit for electric charge is Coulomb [C]. One elementary charged particle has a charge q_0 :

$$q_0 = 1.6021.10^{-19} C$$

When in a given space region are grouped numerous electric charges (positive and negative), the total charge Q in the volume τ is:

$$Q_\tau = \sum q^+ + \sum q^-$$

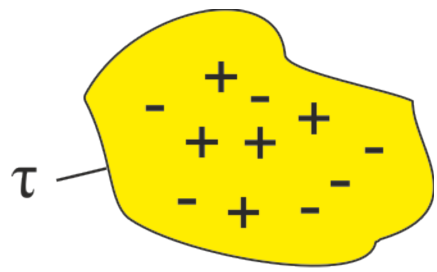


Fig. 1.2.

1.1.2. Electric field and electrostatics

The electric field exists in the space near electrically charged particles and near varying magnetic field. When the electric charges are not moving, such field is called electrostatic. The electric field is a vector whose direction is defined as the direction that a positive test charge would be pushed when placed in a field (fig. 1.3). In other words, the electric field is directed away from positively charged particles and towards negatively charged particles.

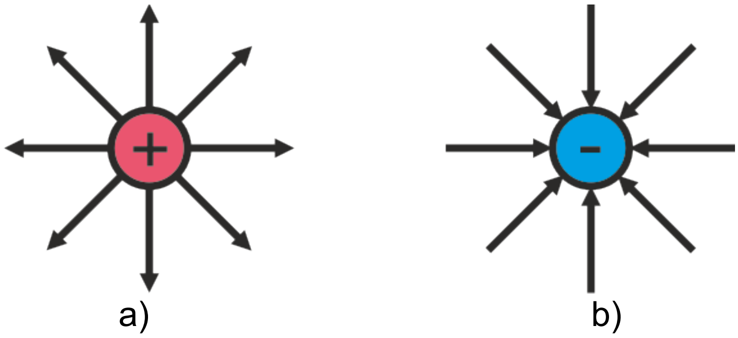


Fig. 1.3.

When differently charged particles are close to each other there is a force of attraction between them (fig. 1.4a). When same charged particles (positive-positive or negative-negative) are close to each other they repel each other (fig. 1.4b). The electric lines around electrically charged particles form the electric field.

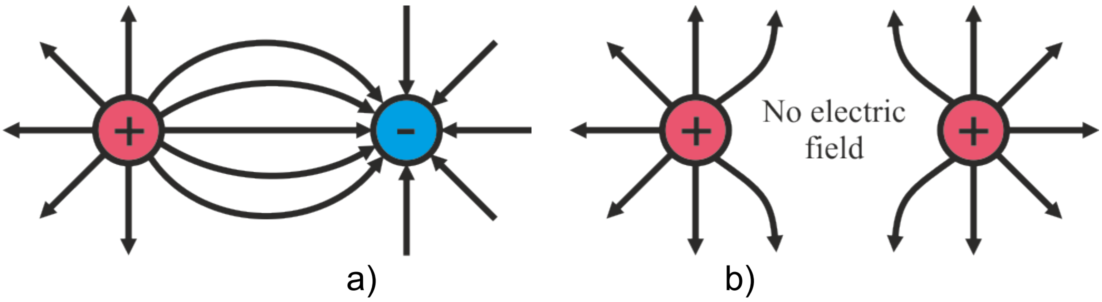


Fig. 1.4. Electrostatic field between charges with: a) the different charge; b) the same charge.

The force $\overrightarrow{F_{12}}$ between two electric charges q_1 and q_2 , at a distance r away from each other, is given with Coulomb's law (fug. 1.5).

$$\overrightarrow{F_{12}} = \frac{q_1 . q_2}{\varepsilon . 4 . \pi . r^2} \overrightarrow{r_{12}}$$

where ε is the electric permittivity of the medium;

$\overrightarrow{r_{12}}$ is a vector with value 1, who's direction shows the direction of the force $\overrightarrow{F_{12}}$.

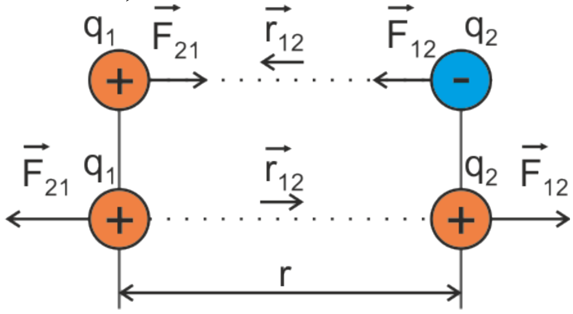


Fig. 1.5.

Consider that a force $\Delta \vec{F}$ is acting on a positive charge ΔQ . The intensity of the electric field (often called simply electric field) $\vec{E}$ is defined as:

$$\vec{E} = \lim_{\Delta Q \to 0} \frac{\Delta \vec{F}}{\Delta Q}$$

The unit for electric field intensity is Volt per meter [$V . m^{-1}$]. The direction of the vector of the electric field $\vec{E}$ coincides with the force $\vec{F}$.

1.1.3. Electric voltage and electric potential

Consider an electric charge is inside an electric field $\vec{E}$ and a force $\vec{F}$ is acting on it (fig. 1.6). Under the influence of this force the charge moves which means that some work is done (some energy is used).

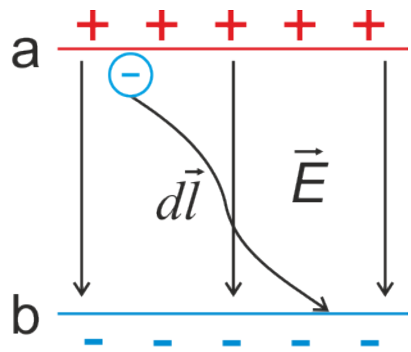


Fig. 1.6.

The energy used to move the charge from point a to point b is:

$$A=\int_a^b \vec{F}.d\vec{l}=q.\int_a^b \vec{E}.d\vec{l}=q.V_{ab}$$

where the vector $d\vec{l}$ follows the trajectory of the moving charge.

$V_{ab}=\int_a^b \vec{E}.d\vec{l}$ is called electric voltage.

The unit for electric voltage is Volt [V].
The electric voltage is a scalar quantity and could have positive or negative sign, depending on the direction of integration.

$$V_{ab}=\int_a^b \vec{E}.d\vec{l}=-\int_b^a \vec{E}.d\vec{l}$$

The above equation shows that a charge increases its potential energy when moving from points a to b, however if the charge then needs to move in the opposite direction it would do it at the expense of its own potential energy.
Consider an electric charge which, under the influence of an electric field, moves from point a to point b and then back to point a (fig. 1.7). The used energy is:

$$A=q.\int_a^b \vec{E}.d\vec{l}+q.\int_b^a \vec{E}.d\vec{l}=q.\int_a^b \vec{E}.d\vec{l}-q.\int_a^b \vec{E}.d\vec{l}=0$$

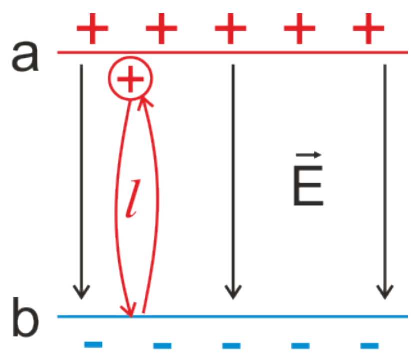


Fig. 1.7.

The above equation shows that the work done by the electric field does not depend on the trajectory of the electric charges but only on the starting and ending points.
Such fields are called conservative fields and allow to define the quantity electric potential (also will be called electric node voltage) as the work required to move a charge from a certain point at infinite distance. The electric potentials of points a and b are:

$$V_a=\int_a^\infty \vec{E}.d\vec{l}$$

$$V_b = \int_b^{\infty} \vec{E}.d\vec{l}$$

The difference between the two potentials gives:

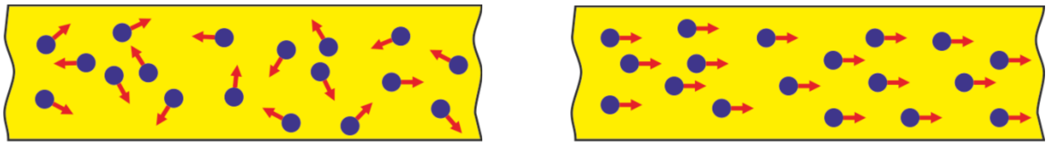
$$V_a - V_b = \int_a^{\infty} \vec{E}.d\vec{l} - \int_b^{\infty} \vec{E}.d\vec{l} = \int_a^{b} \vec{E}.d\vec{l} = V_{ab}$$

In other words the electric voltage V_{ab} can be defined as the difference of the node voltages at points a and b.

1.1.4. Electric current

When no electric field is applied to a conducting material there are free electric charges which move chaotically and their resulting current is 0 (fig. 1.8a). However if an electric current is applied on the conductor, all the free electric charges start moving in the direction of the field (fig. 1.8a):

- Positive charges move towards the negative sign of the electric field;
- Negative charges move towards the positive sing of the electric field.



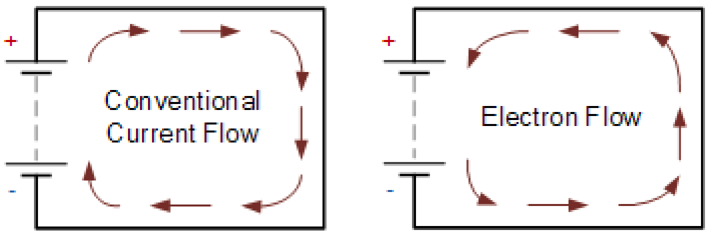
a) b)

Fig. 1.8. Electric charges without an electric field (a) and when an electric field is applied (b)

Any directed motion of electric charges is called electric current. Depending on the state of the substance it is caused by:

- In solid state it is caused by free electrons (negative charges);
- In liquid state it is caused by positive and negative ions;
- In gaseous state it is caused by both ions and electrons.

It should be noted that electric current and the electric charges don't always flow in the same direction. The conventional current flow is from positive to negative voltage potential (fig. 1.9a). However in solid state substances, where the electric charges are electrons, the charge flow is in the opposite direction (fig. 1.9b).



a) b)

Fig. 1.9. Current flow (a) and electron flow (b).

The electric current is described with two quantities:

- Current intensity;
- Current density.

The intensity of electric current (often called simply current) is the electric charge going through a cross-section of the conductor for a given time interval (fig. 1.10):

$$I = \frac{dQ}{dt}$$

If the current is constant in time the above equation becomes:

$$I = \frac{\Delta Q}{\Delta t}$$

The unit for current intensity is Amper [A]. It is a principle unit in the System of units (SI) and the Coulomb unit can be expressed with it:

$$1C = 1A.1s$$

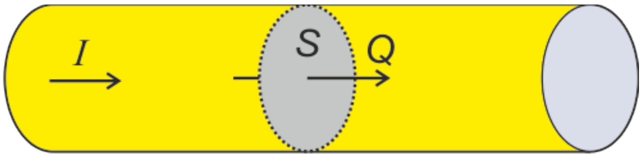


Fig. 1.10.

The current density $\vec{J}$ is a vector quantity, whose value is the current I flowing through a surface S :

$$J = \frac{dI}{dS}$$

The direction of the vector is the same as the direction of the current.
The unit for current density is Amper per square meter [$A.m^{-2}$].

1.2. Electrical networks

An electrical network (circuits) is a device which transforms and distributes electromagnetic energy or information with electric current. The electric circuits could be:

- With concentrated or distributed parameters. Distributed networks are used for transfer of energy/information at a long distance;
- For direct current (DC) or for alternating current (AC);
- Linear or nonlinear.

For an electric circuit the following topology elements could be defined (fig. 1.11):

- A **Node** is where there is a connection between at least three branches;
- A **Branch** is part of the circuit between two consecutive nodes where the current is the same;
- A **Mesh** (or **loop**) is a closed path inside the network.

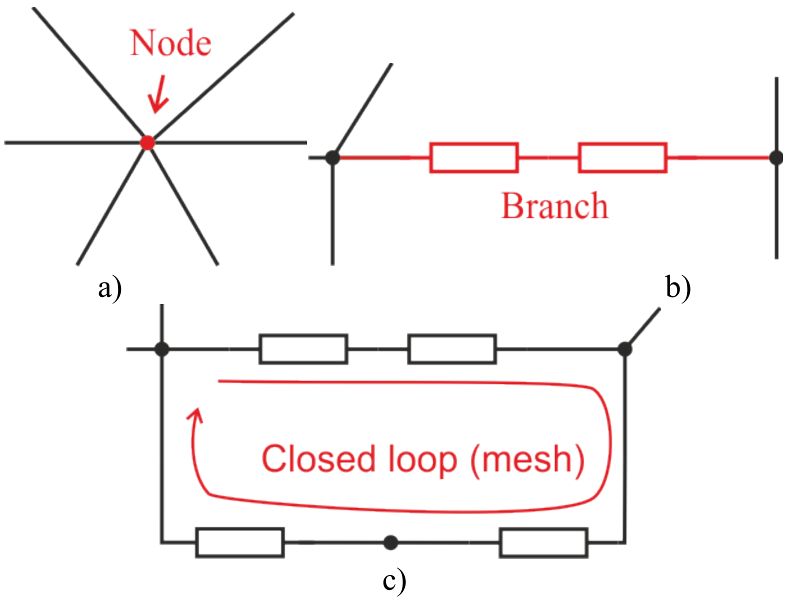


Fig. 1.11. A node (a), a branch (b) and a loop (c).

The basic elements of an electric circuit are:

- Passive elements (consumers) – where the electrical energy is distrubuted (consumed or stored);

- Active elements (energy sources) – they provide the electric energy;
- Connecting wires.

1.2.1. Resistors

The resistor is a passive one-port (two-terminal) element which is used to represent an ideal energy consumer which converts the electrical energy into some other type of energy. Examples for a resistor are a light bulb (the energy is converted to heat and electromagnetic energy), a stove (the energy is converted mainly to heat), etc.
The symbols used to denote a resistor are presented in fig. 1.12.

Fig. 1.12. Resistor symbols.

It is characterized with a resistance R in Ohms or its reciprocal value – conductance in Siemens.

Resistors in series

When resistors are connected in series, their equivallent resistance is the sum of their resistances:

$$R_E = \sum R_k$$

Example: Estimate the equivallent resistance for the circuit in fig. 1.13.

$$R_E = R_1 + R_2 + R_3 = 10 + 20 + 30 = 60\,\Omega$$

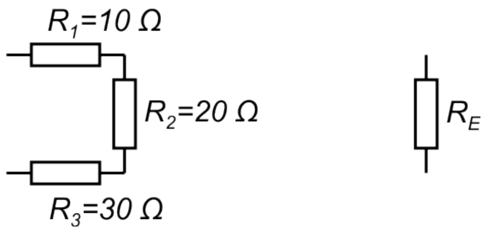


Fig. 1.13.

Resistors in parallel

The equivalent resistance of resistors connected in parallel is estimated with:

$$R_E = \frac{1}{G_E} = \sum \frac{1}{\frac{1}{R_k}} = \sum \frac{1}{G_k}$$

The above equation could also be written for the conductances of the resistors:

$$G_E = \sum G_k$$

Example: Estimate the equivallent resistance for the circuit in fig. 1.14:

$$R_E = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{10} + \frac{1}{20} + \frac{1}{30}} = 5.45\,\Omega$$

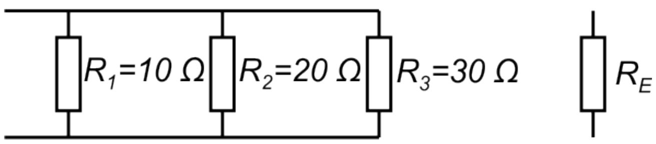


Fig. 1.14.

For two parallel resistors the above equation could be simplified to:

$$R_E = \frac{R_1 . R_2}{R_1 + R_2}$$

For more complex connections of resistors the equivallent resistance could be estimated by by first determining the series and then the parallel resistances.

Example: Estimate the equivallent resistance for the circuit in fig. 1.15.

$$R_E=\frac{(R_2+R_4).R_3}{R_2+R_3+R_4}+R_1=\frac{(20+40).30}{20+30+40}+10=30\,\Omega$$

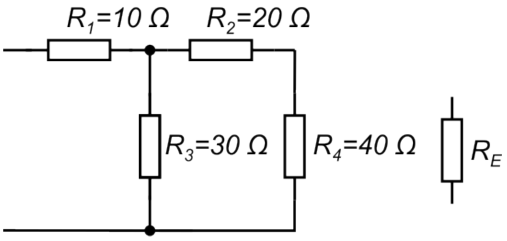


Fig. 1.15.

Resistors connected in star (wye) could be transformed to resistors connected in delta and vice versa (fig. 1.16).

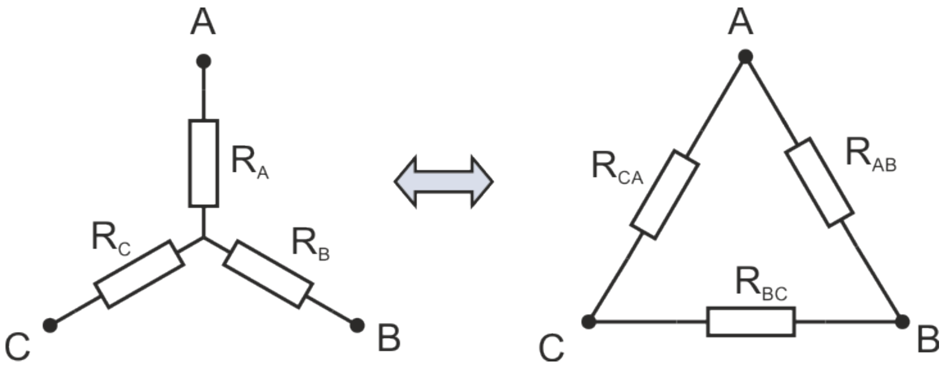


Fig. 1.16.

Star-Delta transformation

$$R_{AB}=R_A+R_B+\frac{R_A R_B}{R_C}$$

$$R_{BC}=R_B+R_C+\frac{R_B R_C}{R_A}$$

$$R_{CA}=R_C+R_A+\frac{R_C R_A}{R_B}$$

Delta-star transformation

$$R_A=\frac{R_{AB}.R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$R_B=\frac{R_{AB}.R_{BC}}{R_{AB}+R_{BC}+R_{CA}}$$

$$R_C=\frac{R_{BC}.R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

1.2.2. Energy sources

Ideal voltage source

An ideal voltage source is a two-terminal device which transforms non-electrical energy to electrical. It creates an electromotive force (emf), measured in Vots.

The symbol for ideal voltage source is presented in fig. 1.17. The last of them is used for DC voltage sources.

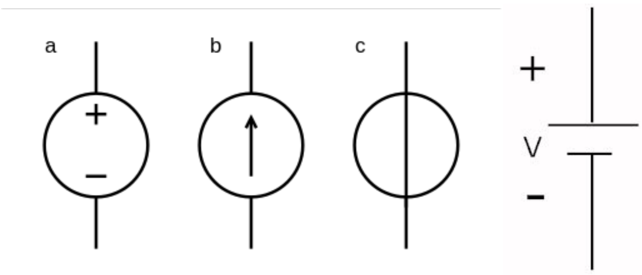


Fig. 1.17. Voltage source symbols.

The plus (+) side of the source has higher potential than the minus (-) one and the created voltage at

the two terminals is equal to the emf.
When voltage sources are connected in series, the equivallent voltage of the source is estimated as the algabraeic sum of the sources:

$$V_E = \sum V_k$$

V_k is positive if the direction of the source is the same as the one of the equivallent source and negative – if it's in the opposite direction. Ideal voltage sources cannot be connected in parallel.
Example: Estimate the equivallent voltage for the soruces in fig. 1.18.

$$V_E = V_1 - V_2 + V_3 = 1.5 - 1.5 + 1.5 = 1.5V$$

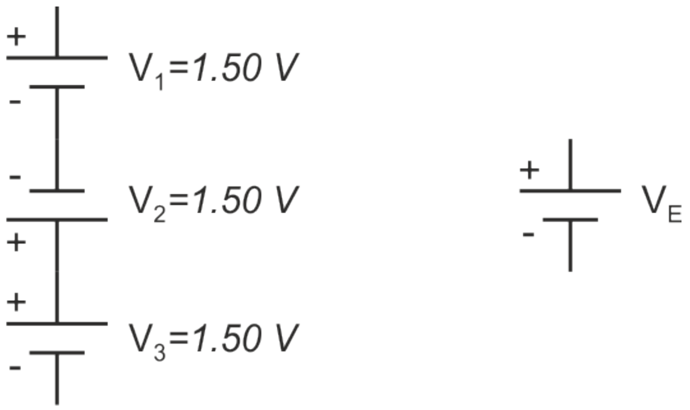


Fig. 1.18.

Real voltage source

A real voltage source is presented as an ideal voltage source and a source resistance, connected in series (fig. 1.19). The output voltage of the source is less than the voltage of the ideal source:
$V_{ab} = V_{SRC} - V_{RSRC}$

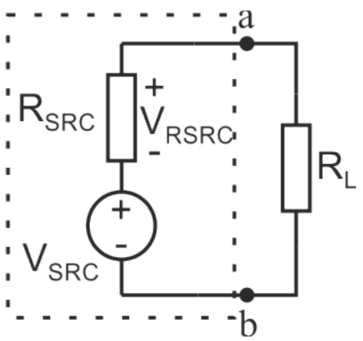


Fig. 1.19. Real voltage source

Ideal current source

An ideal current source is also a device which transforms nonelectric energy into electric, which generates current with intensity I . The ideal current source has zero conductance (infinite resistance).
The symbols used for denotion of current source are presented in fig. 1.20.

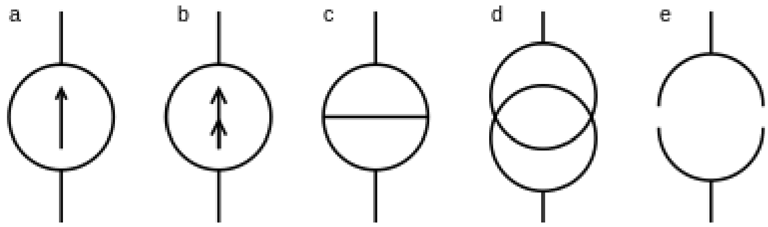


Fig 1.20. Ideal current source.

The ideal current sources could be connected in parallel only and the equivallent current source is estimated as the algebraic sum of the sources.

$$I_E = \sum I_k$$

If I_k has the same direction as I_E, it is added with plus sign and otherwise – with minus. Ideal current sources cannot be connected in series.

Example: Estimate the equivalent current for the sources in fig. 1.21.

$$I_E = I_1 - I_2 + I_3 = 1 - 2 + 3 = 2\,A$$

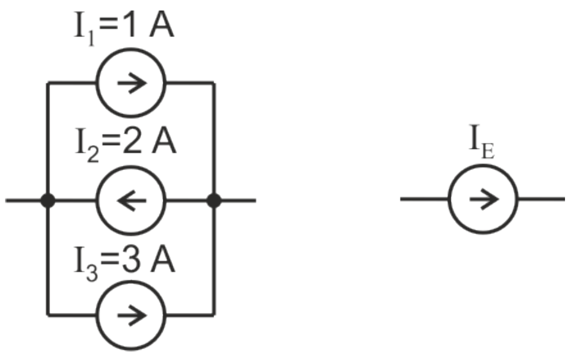


Fig. 1.21.

Real current source

A real current source is presented as ideal current source and an internal conductance (resistance) connected in parallel (fig. 1.22).

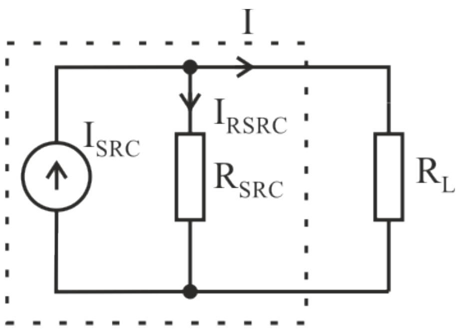


Fig. 1.22.

The current generated by a real current source is less than the current of the ideal source:

$$I_R = I - I_{RSRC}$$

Duality

The duality in electric circuits allow to replace any real voltage source with a real current source and vice versa (fig. 1.23). The internal resistance of both the real sources is R_{SRC} and the equivallent voltage/current are estimated with:

$$V_{SRC} = I_{SRC} . R_{SRC}$$

$$I_{SRC}=\frac{V_{SRC}}{R_{SRC}}$$

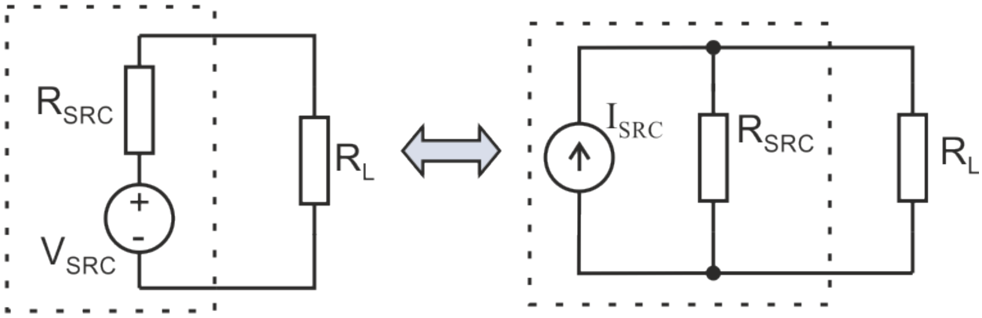


Fig. 1.23.

Dependent sources

In the theory of electrical networks, a dependent source is a voltage or current source, whose value depends on a voltage or current somewhere else in the network. They are widely used to simplify the analysis of electric circuits which include mutually coupled inductors, operational amplifiers, transistors, etc.

The circuit symbols for dependent sources are presented in fig. 1.24.

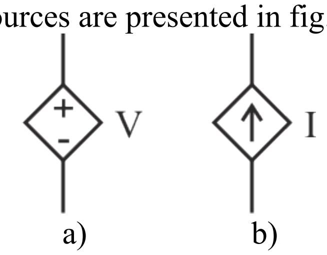


Fig. 1.24. Dependent voltage source (a) and dependent current source (b).

Linear dependent sources can be classified as follows:

- Voltage-controlled voltage source - the source voltage depends on the voltage of another element of the circuit.

$$v=f(v_x)=A_V . v_x$$

where A_V is a coefficient in dimensionless units.

- Current-controlled voltage source - the source voltage depends on the current of another element of the circuit.

$$v=f(i_x)=A_R . i_x$$

where A_R is a resistance coefficient in Ohms.

- Voltage-controlled current source – the source current depends on the voltage of another element of the circuit.

$$i=f(v_x)=A_G . v_x$$

where A_G is a conductance coefficient in Siemens.

- Current-controlled current source – the source current depends on the current of another element of the circui.

$$I=f(i_x)=A_I . i_x$$

where A_I is a coefficient in dimensionless units.

1.3. Main laws in DC circuits

1.3.1. Ohm’s law

The relation between the current I and voltage V of a conductor is given with Ohm’s law (fig. 1.25):

$$I = \frac{V}{R} = V \, . \, G$$

where R is the electric resistance in Ohms [Ω];

$G = \frac{1}{R}$ is the electric conductance, measured in Siemens [S].

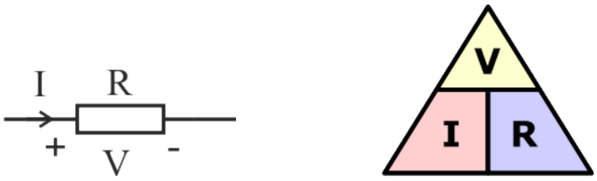


Fig. 1.25.

The quantities electric resistance and conductance describe the material's ability to conduct electric current.

1.3.2. Kirchhoff's circuit laws

For simple circuits containing only 1 source all analysis could be done with equivallent resistance and Ohm's law. However in more complex circuits containing more than one source, Ohm's law cannot be directly applied. For such circuits Gustav Kirchhoff developed a pair of rules, which are known as Kirchhoff's currnet and voltage laws.

Kirchhoff's first law – the current law (KCL)

The Kirchoff's current law or KCL, refers to the current flow of a node (junction). According to KCL the total current (or charge) entering a node is equal to the total current (or charge) leaving the node:

$$\sum I_{IN} = \sum I_{OUT}$$

In other words the total current flow in and out of the node must be equal to zero:

$$\sum I_k = 0$$

Example: Write down the KCL for the node in fig. 1.26:

The solution is:

$$I_1 + I_2 + I_3 = I_4 + I_5$$

or

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

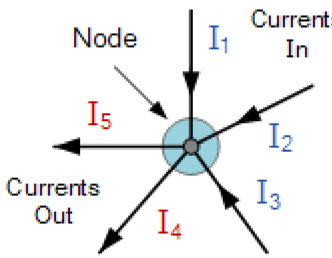


Fig. 1.26.

Kirchhoff's second law – the voltage law (KVL)

The Kirchhoff's voltage low or KVL, refers to any closed loop in the network. It states that the total voltage around the loop is equal to the sum of the voltage drops within the loop:

$$\sum V_{SRC_k} = \sum V_k$$

In other words the sum of all voltage drops around a closed loop is equal to zero:

$$\sum V_k = 0$$

By applying Ohm's law in the above equation it becomes:

$$\sum V_{SRC_k} = \sum V_k = \sum R_k \, . \, I_k$$

Example: Write down the KVL for the loop in fig. 1.27 :

$$V_{AB}+V_{BC}+V_{CD}+V_{DA}=0$$

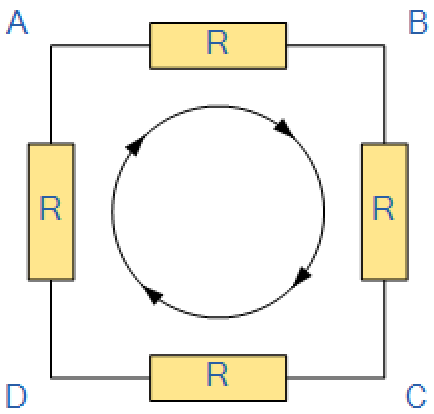


Fig. 1.27.

Example: Write down KCL and KVL for all nodes and loops of the circuit in fig. 1.28.

The KCL for node A

$$I_1+I_2=I_3$$

The KCL for node B

$$I_3=I_1+I_2$$

The KVL for loop 1

$$V_1=V_{R1}+V_{R3}=I_1.R_1+I_3.R_3$$

The KVL for loop 2

$$V_2=V_{R2}+V_{R3}=I_2.R_2+I_3.R_3$$

The KVL for loop 3

$$V_1-V_2=V_{R1}-V_{R2}=I_1.R_1-I_2.R_2$$

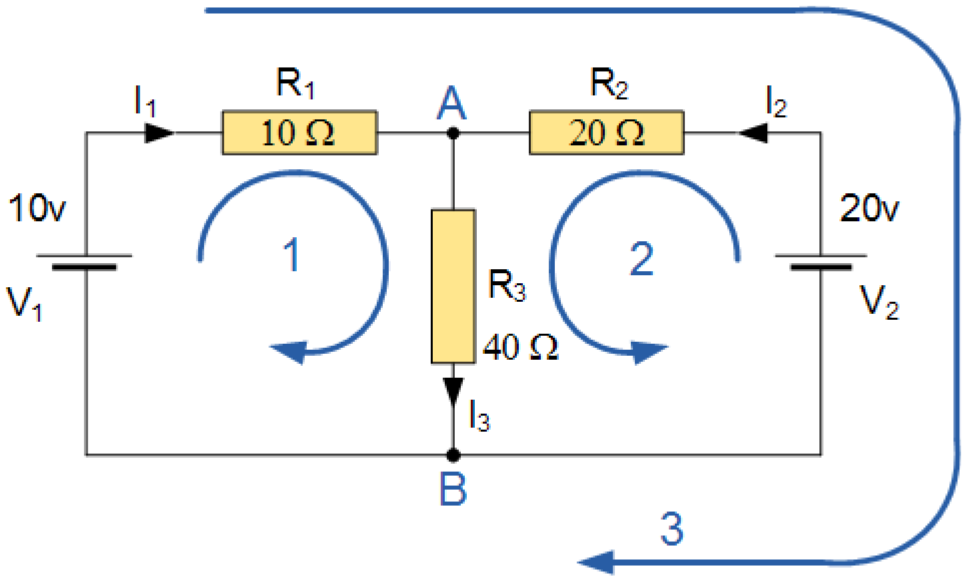


Fig. 1.28.

Example: Voltage divider

Consider the circuit in fig. 1.29. He same current flows through both resistors, so the voltage ratio between V_{R1} and V_{R2} is equal to the resistance ratios.

$$\frac{V_{R1}}{V_{R2}}=\frac{I.R_1}{I.R_2}=\frac{R_1}{R_2}$$

The KVL for this circuit is:

$$V = V_{R1} + V_{R2}$$

This means that the source voltage is divided between the two resistors at a ratio:

$$\frac{V_{R1}}{V_{R2}} = \frac{R_1}{R_2} = \frac{10}{20} = \frac{1}{2}$$

or

$$V_{R1} = \frac{R_1}{R_1 + R_2}.V = \frac{1}{3}.12 = 4\,V$$

$$V_{R2} = \frac{R_2}{R_1 + R_2}.V = \frac{2}{3}.12 = 8\,V$$

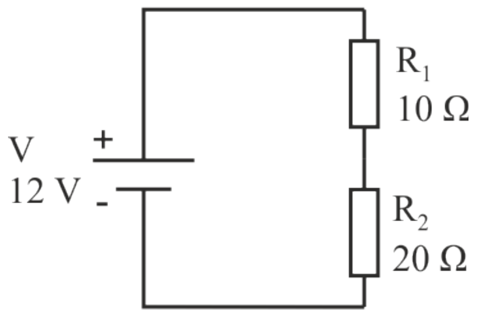


Fig. 1.29.

Example: Current divider.

Consider the circuit in fig. 1.30. The voltage drop on R_1 and R_2 is the same since the elements are connected in parallel. Then according to Ohm's law the current ratio between of the two branches is:

$$\frac{I_1}{I_2} = \frac{\frac{V_{12}}{R_1}}{\frac{V_{12}}{R_2}} = \frac{\frac{1}{R_1}}{\frac{1}{R_2}} = \frac{R_2}{R_1} = \frac{40}{10} = \frac{4}{1}$$

The following KCL could be written for the circuit:

$$I = I_1 + I_2$$

This means that the current I divides into I_1 and I_2 according to:

$$I_1 = \frac{R_2}{R_1 + R_2} I = \frac{40}{10+40}.1 = 0.8\,A$$

$$I_2 = \frac{R_1}{R_1 + R_2} I = \frac{10}{10+40}.1 = 0.2\,A$$

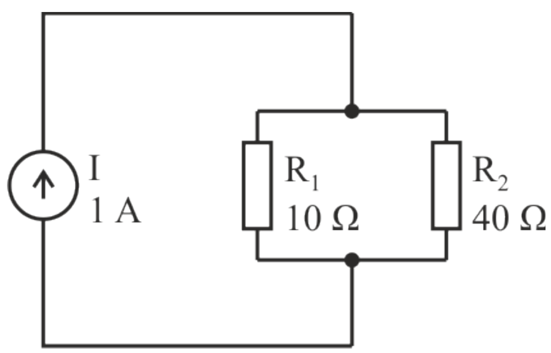


Fig. 1.30.

1.3.3. Power

The power P dissipated in a conductor as heat can be estimated according to Joule's law:

$$P=V.I$$

where V and I are the voltage drop and the current flow through the conductor.
By applying Ohm's law in the above equation the power could also be expressed as:

$$P=V.I=R.I.I=I^2.R$$

$$P=V.I=V.\frac{V}{R}=\frac{V^2}{R}$$

The unit for power is Watt [W] and it represents the energy consumption per unit of time:

$$1W=\frac{1J}{1s}$$

Maximum power transfer theorem

Consider the circuit in fig. 1.31. A real voltage source with voltage V_s and output resistance R_s is powering a load R_L . The current of the circuit is:

$$I=\frac{V_s}{R_s+R_L}$$

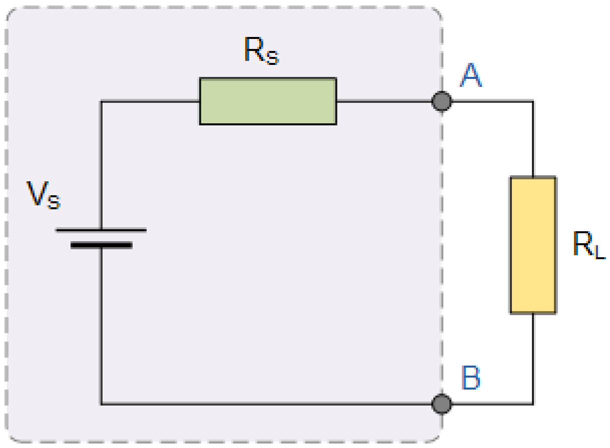


Fig. 1.31.

Then the power delivered to the load is:

$$P_{RL}=I^2.R_L=\left(\frac{V_s}{R_s+R_L}\right)^2 R_L=V_s^2.\frac{R_L}{R_s^2+2.R_s.R_L+R_s^2}=\frac{V_s^2}{\frac{R_s^2}{R_L}+\frac{2.R_S.R_L}{R_L}+\frac{R_L^2}{R_L}}=\frac{V_s^2}{\frac{R_s^2}{R_L}+2.R_S+R_L}$$

To maximize the power output, $\frac{R_s^2}{R_L}+2.R_S+R_L$ should be minimal. This is fulfiled when its first derivative is zero:

$$\frac{d\left(\frac{R_s^2}{R_L}+2.R_S+R_L\right)}{dR_L}=\frac{-R_s^2}{R_L^2}+1=0$$

The condition then becomes:

$$\rightarrow\frac{R_s^2}{R_L^2}=1$$

or

$$R_L=R_S$$

A source with output resistance R_S will deliver maximal power to a load R_L if $R_L=R_S$.

Conservation of power

The conservaiton of power in electrical circuit reflects the conservation of energy law. According to it the power delivered by the sources is equal to the power consumption in the resistors of the circuit:

$$\sum P_{SRC_k} = \sum P_k$$

where P_{SRC_k} is the power delivered by the k-th source;
P_k is the power consumed by the k-th resistor.

By applying Joule's law, the above equation could be written as:

$$\sum V_{SRC_k} . I_{SRC_k} = \sum I_k^2 . R_k$$

- For voltage sources V_{SRC_k} is the voltage of the source and I_{SRC_k} – the current through the source.
- For current sources I_{SRC_k} is the current of the source and V_{SRC_k} – the voltage drop on the source.

1.4. Methods and theorems for analysis of linear DC circuits

For analysis of linear electrical networks the following data is known:

- The circuit schematics;
- The voltage of the voltage sources;
- The current of the current sources;
- The resistances of the consumers.

The goal of the anlaysis is to estimate all currents and powers in the circuit.

1.4.1. Kirchhoff's laws analysis

The most universal method for analysis of electric networks is to directly apply Kirchhoff's voltage and current laws. It is applicable for both linear and nonlinear networks. The method includes:

1. Choose the conditional branch currents directions (if not shown on the figure);
2. Estimate the number of KCL equations as the number of nodes minus 1;
3. Estimate the number of KVL equations as the number of unknown currents minus the number of the KCL requations;
4. Write down and solve a system of linear equations.
5. **Limitations:** With this method KVL loops can be closed through a current source.

Example: Estimate the currents for the circuit in fig. 1.32. using Kirchhoff's laws and verify the power conservation.

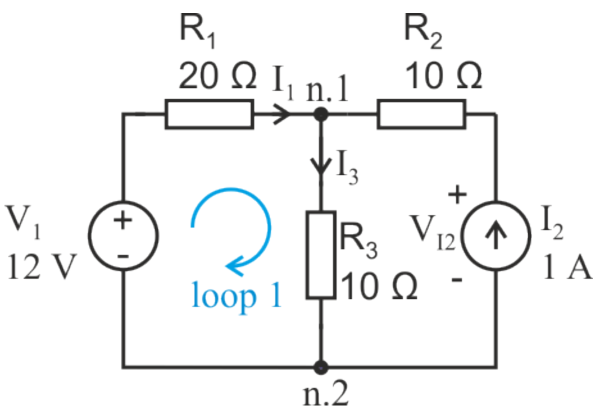


Fig. 1.32.

1. The circuit includes 2 nodes, which means 1 KCL equation could be written;
2. The circuit includes 3 branches and 1 current source, which means there are 2 unknown currents. Then the number of KVL equations is 1.

$$\left| \begin{array}{c} I_1+I_2=I_3 \\ V_1=I_1.R_1+I_3.R_3 \end{array} \right. \rightarrow \left| \begin{array}{c} 1=-I_1+I_3 \\ 12=20.I_1+10.I_3 \end{array} \right.$$

The above system could be written in matrix form as:

$$\begin{vmatrix} I_1 \\ I_3 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ 20 & 10 \end{vmatrix} = \begin{vmatrix} 1 \\ 12 \end{vmatrix}$$

The determinants are:

$$\Delta = Det \begin{vmatrix} -1 & 1 \\ 20 & 10 \end{vmatrix} = -1.10-1.20=-30$$

$$\Delta_1 = Det \begin{vmatrix} 1 & 1 \\ 12 & 10 \end{vmatrix} = 1.10-1.12=-2$$

$$\Delta_3 = Det \begin{vmatrix} -1 & 1 \\ 20 & 12 \end{vmatrix} = -1.12-1.20=-32$$

Then the currents are:

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2}{30} = 0.067\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{32}{30} = 1.067\,A$$

The power of the voltage source is:

$$P_{V1} = V_1 . I_1 = 12 * 0.067 = 0.804\,W$$

The power of the current source is:

$$P_{I3} = V_{I2} . I_2$$

The voltage drop V_{I2} is unknown and could be estimated by writing a KVL for a loop through the current source I_2 .

$$V_{I2} = I_2 . R_2 + I_3 . R_3 = 1*10 + 1.067*10 = 20.67\,V$$

$$P_{I3} = V_{I2} . I_2 = 20.67*1 = 20.67\,W$$

Then the total source power is:

$$P_{SRC} = P_{V1} + V_{I2} = 0.804 + 20.67 = 21.474\,W$$

The consumed power is:

$$P_{CONS} = I_1^2 . R_1 + I_2^2 . R_2 + I_3^2 . R_3 = 0.067^2 .20 + 1.067^2 .10 + 1^2 .10 = 21.474\,W$$

$P_{SRC} = P_{CONS}$ which means that the circuit analysis was performed correctly.

1.4.2. Mesh currents analysis

The mesh current analysis is based on KVL. The goal of this method is to create enough loops inside the circuit so that they cover all elements. These currents are called circullating (or mesh) currents.

The mesh current anlaysis method includes:

- Choose the conditional branch currents directions (if not shown on the figure);
- Choose a number of inside loops in order to cover all elements of the circuit and name them $I_k^{(m)}$. As a general rule only label inside loops in a clockwise direction.
- If a mesh current circulates through a current souce, it is equal to the current source.
- Write down KVL for each of the loops using the mesh currents.
- The branch currents are an algebraic sum of the mesh current going through them:

$$I_k = \sum I_n^{(m)}$$

- Sove the system to find the mesh and branch currents.
- **Limitations:** No mesh current should go through more than one current source. This method is only applicable for linear circuits.

Example: Estimate the currents for the circuit in fig. 1.32. using mesh currents analysis.
The mesh currents in the circuit are presented in fig. 1.33.

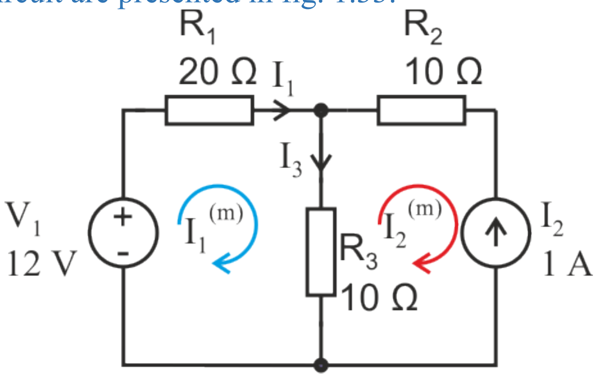


Fig. 1.33.

The mesh current $I_2^{(m)}$ circulates through the current source with an opposite direction, so it's value is $I_2^{(m)}=-I_2=-1\,A$

$$\begin{cases} I_2^{(m)}=-I_2=-1 \\ V_1=R_1 I_1^{(m)}+R_3\left(I_1^{(m)}-I_2^{(m)}\right) \end{cases}$$

Then the mesh current $I_1^{(m)}$ is:

$$I_1^{(m)}=\frac{V_1+I_2^{(m)}R_3}{R_1+R_3}=\frac{12-10}{20+10}=0.067\,A$$

The branch currents are an algebraic sum of the mesh currents circulating through them:

$$I_1=I_1^{(m)}=0.067\,A$$

$$I_3=I_1^{(m)}-I_2^{(m)}=0.067-(-1)=1.076\,A$$

The obtained values of I_1 and I_3 are the same as in the Kirchhoff's laws analysis.

1.4.3. Node voltage analysis

The node voltage analysis is based on KCL. The goal of this method is to write enough equations using the KCL so that all branch currents are included. Then each current is estimated using the node voltages. This method includes:

1. Choose the conditional branch currents directions (if not shown on the figure);
2. Choose one of the nodes as reference node and ground it (I $V_{ref}=0\,V$);
3. If there are n nodes in the circuit, white down n-1 equations using the KCL;
4. Express each branch current with Ohm's law, using the node voltages;
5. The current of a branch with current source is equal to the current source;
6. Write down and solve the equation with node voltages;
7. Estimate the branch currents with the equations from point 4.

Note: It is recommended that the reference node is chosen to be the one which has the most branches connected to;

Limitations: This method is not directly applicable for nonlinear circuits analysis.

Example: Estimate the currents for the circuit in fig. 1.32. using node voltage analysis.
The circuit has two nodes. Node 2 is being connected to ground, so the node voltage $V_{n2}=0\,V$ (fig. 1.34).

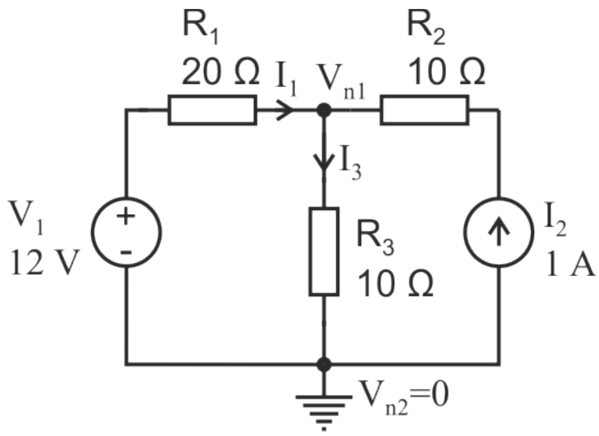


Fig. 1.34.

There is one unknown node voltage (V_{n1}) so one equation is required. The KCL for node 1 is:

$$I_1+I_2=I_3$$

By applying Ohm's law the currents are:

$$I_1=\frac{V_{n2}-V_{n1}+V_1}{R_1}=\frac{-V_{n1}+12}{20}$$

$$I_2=1\,A$$

$$I_3=\frac{V_{n1}-V_{n2}}{R_3}=\frac{V_{n1}}{10}$$

Then the KCL becomes:

$$\frac{-V_{n1}+12}{20}+1=\frac{V_{n1}}{10}=¿-10.V_{n1}+120+20.10=V_{n1}.20$$

$$\rightarrow V_{n1}=\frac{320}{30}=10.67\,V$$

Then the branch currents are:

$$I_1=\frac{-V_{n1}+12}{20}=\frac{-10.67+12}{20}=0.067\,A$$

$$I_3=\frac{V_{n1}}{10}=\frac{10.67}{10}=1.067\,A$$

The obtained values of I_1 and I_3 are the same as in the other methods.

1.4.4. Thevenin's and Norton's theorems

Thevenin's theorem states that any linear network containing multiple sources and resistances, which can be considered a one port (two terminals) could be replaced with an equivalent real voltage source (fig. 1.35). Norton's theorem is similar and states that any linear one port network could be replaced with an equivalent real current source (fig. 1.36).

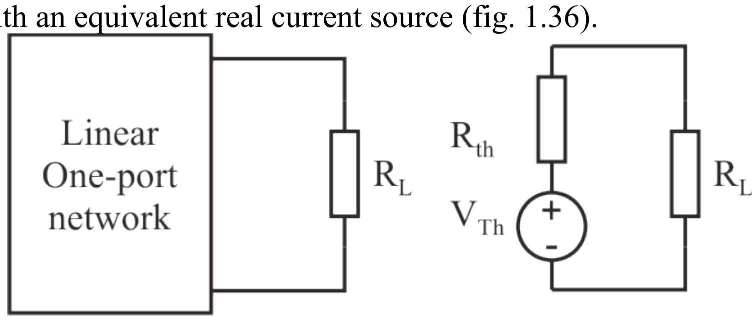


Fig. 1.35.

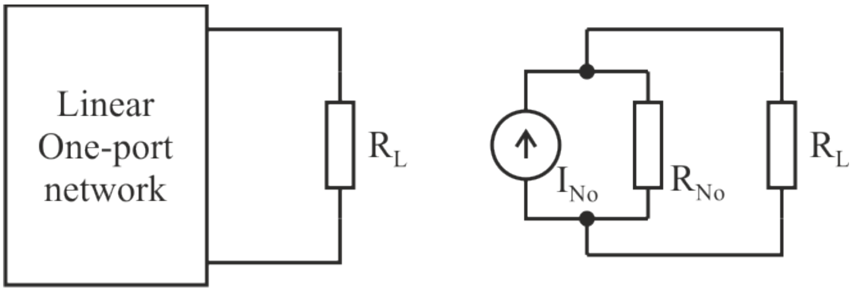


Fig. 1.36.

The Thevenin and Norton equivalent circuits could be estimated using the open circuit voltage, short circuit current and the equivalent output resistance of the one-port (fig. 1.37). When estimating the equivalent output resistance all voltage sources are replaced with a short circuit and all current sources – with an open circuit.

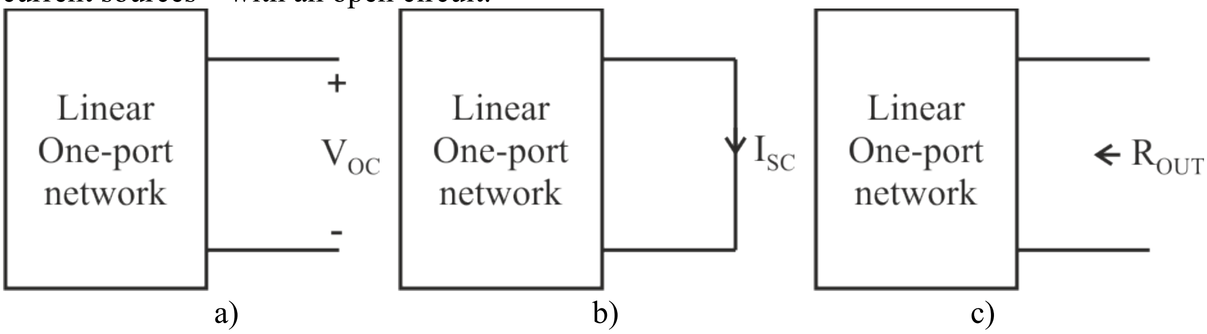


Fig. 1.37. Estimation of the open circuit voltage (a), short circuit current (b) and output resistance (c).

The Thevenin and Norton equivalent circuit parameters are:

$$V_{Th}=V_{OC} \qquad I_{No}=I_{SC} \qquad R_{Th}=R_{No}=R_{OUT}=\frac{V_{OC}}{I_{SC}}$$

Limitations: The theorems are not applicable for nonlinear one-ports. However they are applicable if the nonlinear elements are part of the load.

Example: Estimate the current I_3 for the circuit in fig. 1.31. using Thevenin's equivallent circuit.

In order to estimate the open circuit voltage, the resistor R_3 is removed and the network is solved (fig. 1.38a). The circuit becomes a single loop network with current source, so the current I_1 is:

$$I_1=-I_2=1\,A$$

Then the open circuit voltage could be estimated using the KVL:

$$V_1=R_1.I_1+V_{OC} \rightarrow V_{OC}=12-20.(-1)=32\,V.$$

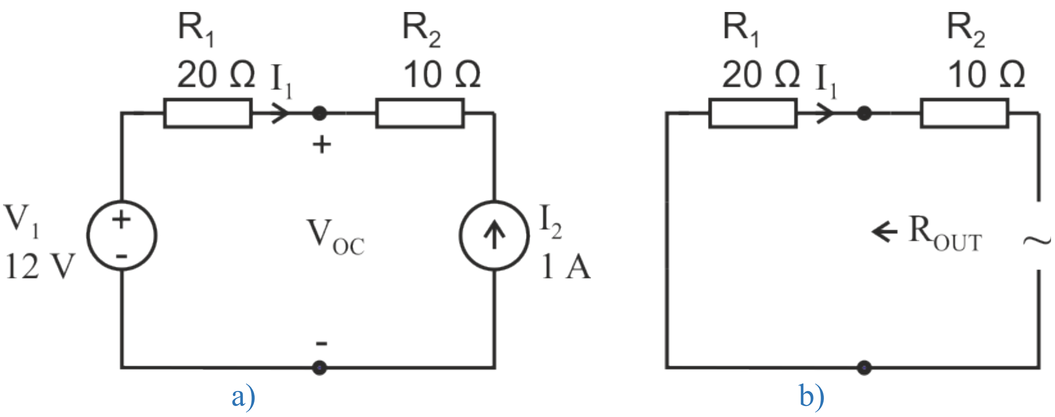


Fig. 1.38. Open circuit (a) and equivalent resistance circuit (b).

The Thevenin's equivalent resistance is estimated from fig. 1.38b. The voltage source is short circuit, and the current source – open circuit:

$$R_{OUT}=R_1=20\,\Omega$$

Then the Thevenin equivalent source is:

$$V_{Th}=V_{OC}=32\,V$$

$$R_{Th}=R_{OUT}=20\,\Omega$$

Thevenin's equivalent circuit is presented on fig. 1.39. The current through R_3 is:

$$I_3=\frac{V_{Th}}{R_{Th}+R_3}=\frac{32}{20+10}=1.067\,A$$

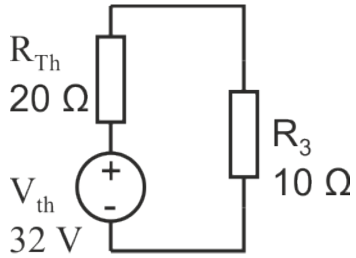


Fig. 1.39. Thevenin's equivalent circuit.

Example: Estimate the current I_3 for the circuit in fig. 1.32. using Norton equivallent circuit. In order to estimate the short circuit current, the resistor R_3 is replaced with a short circuit (fig. 1.40). The short circuit current is:

$$I_{No}=I_{SC}=\frac{V_1}{R_1}+I_2=0.6+1=1.6\,A$$

The Norton equivalent resistance is equal to the Thevenin equivalent resistance $R_{No}=20\,\Omega$. However it could also be obtained with:

$$R_{No}=\frac{V_{OC}}{I_{SC}}=\frac{32}{1.6}=20\,\Omega$$

This method is especially useful when R_{No} needs to be determined experimentally.

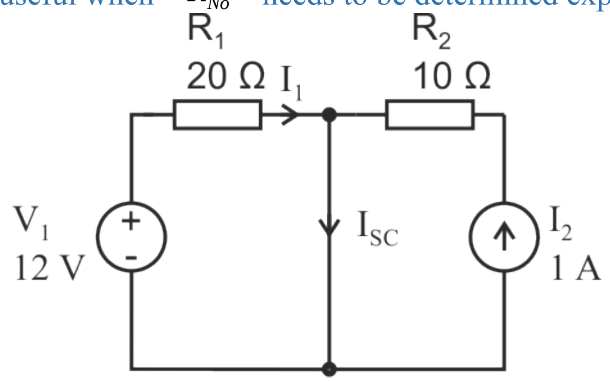


Fig. 1.40. Short circuit at the place of R_3 .

Norton's equivalent circuit is shown in fig. 1.41.
Considering this is current divider, the current I_3 can be estimated with:

$$I_3=I_{No}\cdot\frac{R_{No}}{R_{No}+R_3}=1.6\cdot\frac{20}{20+10}=1.067\,A$$

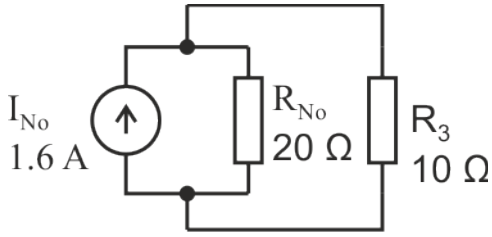


Fig. 1.41. Norton's equivalent circuit.

1.4.5. The superposition theorem

The superposition theorem states that in a linear circuit with several sources the currents and voltages for any element in the circuit is the sum of the currents and voltages produced by each source acting independently. When calculating the contribution of each source all other sources need to be removed:

- Voltage sources are replaced with a short circuit;
- Current sources are replaced with an open circuit.

Example: Estimate the current I_3 for the circuit in fig. 1.32. using the superposition theorem. In order to calculate the contribution of the voltage source V_1, the current source I_2 is replaced with an open circuit (fig. 1.42a). Then the current $I_3^{(V1)}$ becomes:

$$I_3^{(V1)} = \frac{V_1}{R_1 + R_3} = \frac{12}{20+10} = 0.4\,A$$

In order to calculate the contribution of the current source I_2, the voltage source is replaced by a short circuit (fig. 1.42b). Considering this is a current divider, $I_3^{(I_2)}$ becomes:

$$I_3^{(I_2)} = I_2 . \frac{R_1}{R_1 + R_3} = 1 . \frac{20}{20+10} = 0.67\,A$$

Then the current I_3 is the sum of the current produced by the two sources independently:

$$I_3 = I_3^{(V1)} + I_3^{(I_2)} = 0.4 + 0.667 = 1.067\,A$$

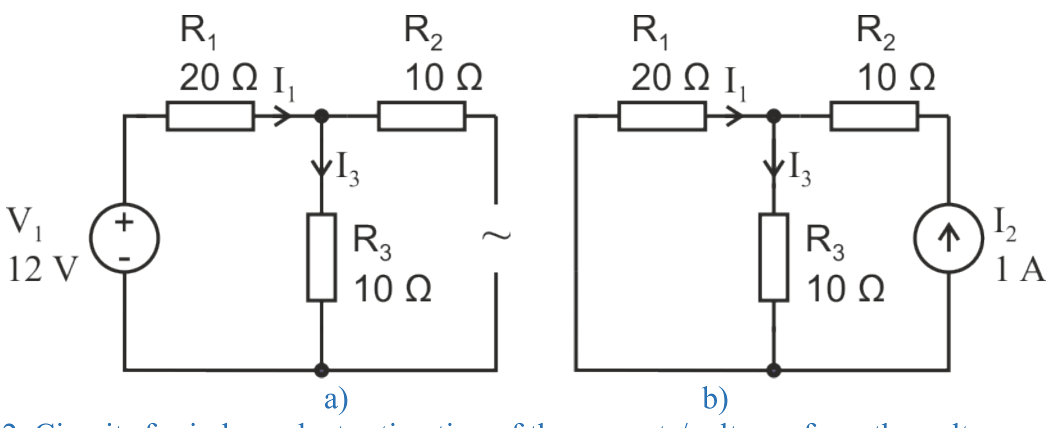


Fig. 1.42. Circuits for independent estimation of the currents/voltages from the voltage source (a) and the current source (b).

References

1. Alexander Ch., Sadiku M. Fundamentals of electric circuits. Fifth edition. McGraw-Hill. 2013. Chapter 1, 2, 3 and 4.
2. http://www.electronics-tutorials.ws/category/dccircuits