LABORATORY EXERCISE 4

Frequency response in resonant circuits.

Goal of the lab: To demonstrate the series and parallel resonance phenomenons in electric circuits.

1. Required equipment

Equipment	Count
Breadboard	1 pc.
A set of connecting wires	1 pc.
Function generator	1 pc.
Two channel oscilloscope	1 pc.
Inductor 290 μH	1 pc.
Capacitor 100 nF	1 pc.
Resistor 100 Ω, 5 W	1 pc.

2. Introduction

The reactances of inductors and capacitors in steady-state sinusoidal circuits depend on the frquency:

$$X_C = \frac{1}{\omega . C}$$

$$X_L = \omega . L$$

where $\omega = 2.\pi.f$ is the angular frequency in $rad.s^{-1}$, C and L are respectively the capacitance of the capacitor in F, and the inductance of the coil in H. The complex impedance Z of a series RLC circuit (Fig. 1) is:

$$Z(\omega) = R + j\omega L - j\frac{1}{\omega C} = R + j(X_L - X_C)$$

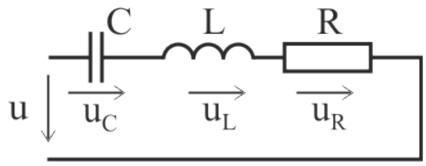


Fig. 1. Series RLC circuit.

The situation when $X_L = X_C$ (or $\omega . L = \frac{1}{\omega . C}$) is called series resonance and it occurs at the resonant frequency ω_0:

$$\omega_0 = \frac{1}{\sqrt{L.C}}$$

Under these conditions, the complex impedance is entirely active:

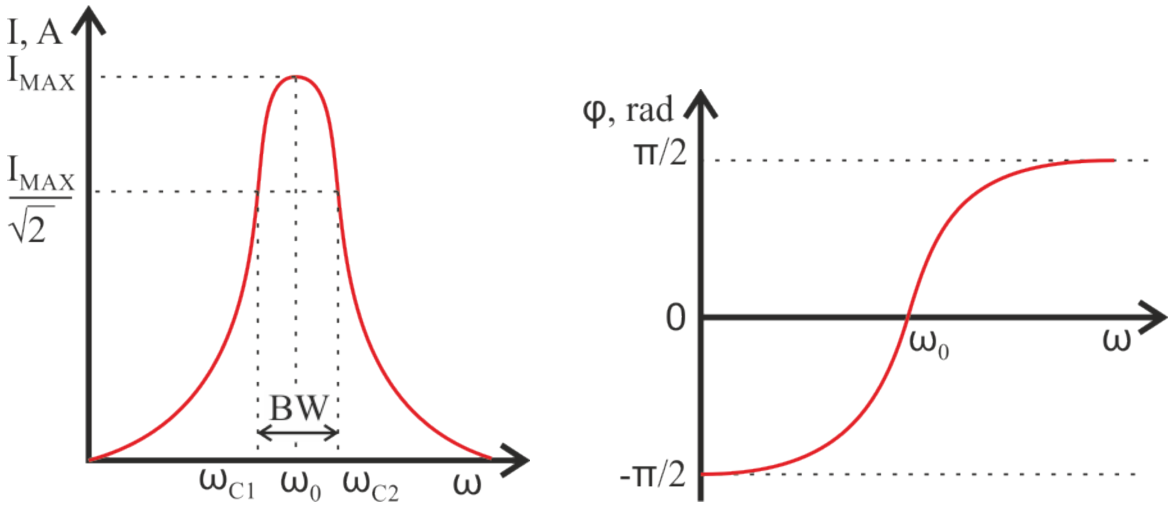
$$Z(\omega) = R + j(X_L - X_C) = R$$

i.e. the inductor and capacitor together are a short circuit, therefore the current in the circuit has a maximum (Fig. 2a):

$$I_{MAX}=\frac{U}{Z(\omega_0)}=\frac{U}{R}$$

For the same reaons, during series resonance, the phase difference φ of the circuit is 0° (Fig. 2b):

$$\varphi=arctg\frac{X_L-X_C}{R}=arctg\frac{0}{R}=0$$



a) b)

Fig. 2. Frequency response of the current $I(\omega)$ (a) and the phase difference $\varphi(\omega)$ (b) in a series RLC circuit.

Similarly, the complex admitance of a parallel RLC circuit (Fig. 3) is:

$$Y=\frac{1}{R}-j\left(\frac{1}{\omega.L}-\omega.C\right)=\frac{1}{R}-j(B_L-B_C)$$

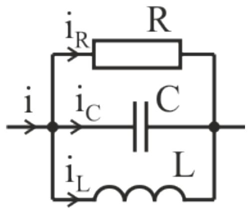


Fig. 3. Paralel RLC circuit.

In this case a parallel resonance could occur, if the following condition is met: $B_L=B_C$ (or $\frac{1}{\omega.L}-\omega.C$); therefore, the condition is the same as in series resonance. When parallel resonance occurs, the complex admitance of the circuit becomes entirely active:

$$Y=\frac{1}{R}-j(B_L-B_C)=\frac{1}{R}$$

therefore the currents i_L and i_C through the inductor and the capacitor are equal, but with opposite directions, i.e.:

$$i_L + i_C = 0$$

This means that the whole input current i will go through the resistor:

$$i = i_R + i_L + i_C = i_R$$

If there is no parallel resistor (parallel RL circuit), the parallel resonance acts as an open circuit and no current will flow.

3. Tasks

Task 1. Investigate the series resonance for the circuit in Fig. 4, with the following elements: $C = 100\,nF$, $L = 290\,\mu H$ and $R = 100\Omega$.

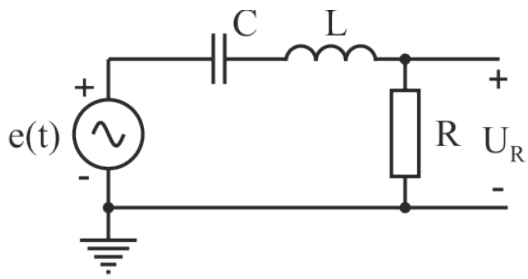


Fig. 4. Circuit for investigation of series resonance.

Step 1. Connect the circuit from Fig. 4 on the breadboard. Connect one of the oscilloscope probes to the source and the other to the resistor.

Step 2. For each frequency in the report:

- Using the oscilloscope readings, set the amplitude of the input voltage to $U_{m.\mathrm{in}} = 2\,V$;
- Measure the amplitude value U_{mR} of the voltage drop on the resistor;
- Estimate the amplitude of the current using Ohm's law:

$$I_{mR} = \frac{U_{mR}}{R}$$

- Measure the phase difference t_φ between the input voltage and the current (the resistor voltage) in seconds (Fig. 5);

Note: If u_{in} leads u_R, the phase difference t_φ is positive (Fig. 5a). Otherwise, it is negative (Fig. 5b).

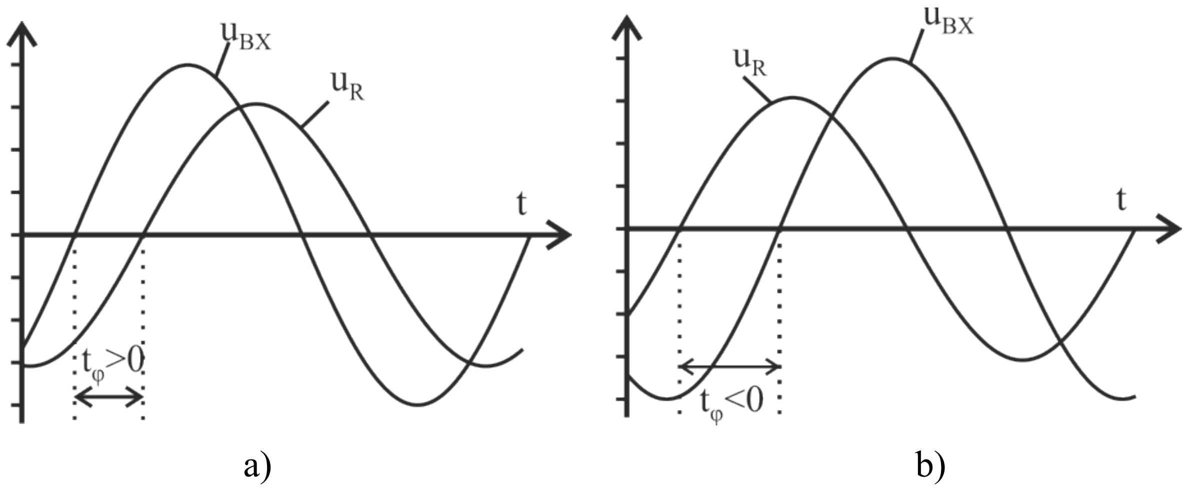


Fig. 5. Measurement of the phase difference in seconds: a) $t_\varphi > 0$; b) $t_\varphi < 0$.

- Estimate the phase difference φ in degrees:

$$\varphi = \frac{360.t_\varphi}{T} = 360.t_\varphi.f$$

Step 3. Draw the frequency response $I_{mR}(f)$.

- Estimate the amplitude of the current, for which only half of the maximal power is consumed:

$$I_{0,5P} = \frac{I_{MAX}}{\sqrt{2}}$$

- Draw the current $I_{0,5P}$ and graphically obtain the cut-off frequencies f_{C1} and f_{C2}, as shown in Fig. 2a.
- Estimate the bandwidth of the circuit:

$$BW = f_{C2} - f_{C1}$$

- Estiamte the quality factor Q of the circuit:

$$Q = \frac{f_0}{BW}$$

Step 4. Draw the frequency response $\varphi(f)$.

<u>Task 2.</u> Investigate the parallel resonance for the circuit in Fig. 6, using the following elements: $C = 100\,nF$, $L = 290\,\mu H$ and $R = 100\,\Omega$.

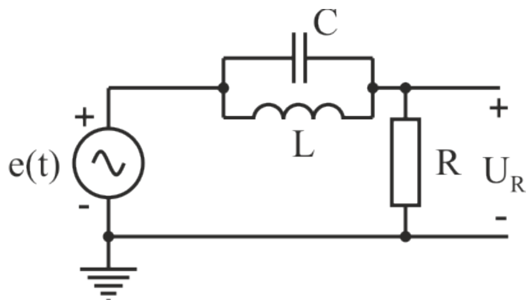


Fig. 6. Circuit for investigation of parallel resonance.

Step 1. Connect the circuit from Fig. 6 on the breadboard. Connect one of the oscilloscope probes to the source and the other to the resistor.

Step 2. For each frequency in the report:

- Using the oscilloscope readings, set the amplitude of the input voltage to $U_{m.\mathrm{in}}=2V$;
- Measure the amplitude value U_{mR} of the voltage drop on the resistor;
- Estimate the amplitude of the current using Ohm's law:

$$I_{mR}=\frac{U_{mR}}{R}$$

Step 3. Draw the frequency response $I_{mR}(f)$.

- Estimate the amplitude of the current, for which only half of the maximal power is consumed:

$$I_{0,5P}=\frac{I_{MAX}}{\sqrt{2}}=\frac{\frac{U_{m.вх}}{R}}{\sqrt{2}}$$

- Draw the current $I_{0,5P}$ and graphically obtain the cut-off frequencies f_{C1} and f_{C2}, as shown in Fig. 2a.
- Estimate the bandwidth of the circuit:

$$BW=f_{C2}-f_{C1}$$

- Estiamte the quality factor Q of the circuit:

$$Q=\frac{f_0}{BW}$$

4. Questions

1. What is the reason for the frequency response of circuits with inductors and capacitors?

2. What happens when series resonance occurs?

3. What happens when parallel resonance occurs?