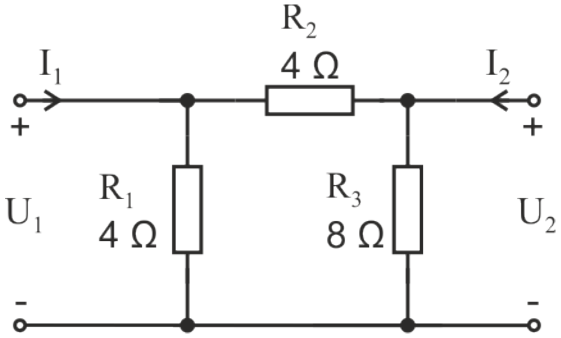
TWO-PORT NETWORKS ANALYSIS

Problem 1. Obtain the hybrid (H) parameters of the two-port.



The system with the H parameters is:

$$\begin{cases}\dot{U}_1=H_{11}\dot{I}_1+H_{12}\dot{U}_2\\ \dot{I}_2=H_{21}\dot{I}_1+H_{22}\dot{U}_2\end{cases}$$

In order to obtain them we need to analyze the circuit once for $I_1=0$ and once for $U_2=0$.

For $I_1=0$ the system of H equations becomes:

$$\begin{vmatrix}U_1=H_{11}I_1+H_{12}U_2\\ I_2=H_{21}I_1+H_{22}U_2\end{vmatrix}\rightarrow\begin{vmatrix}U_1=0+H_{12}U_2\\ I_2=0+H_{22}U_2\end{vmatrix}$$

Then two of the hybrid parameters are:

$$H_{12}=\frac{U_1}{U_2}\,H_{22}=\frac{I_2}{U_2}$$

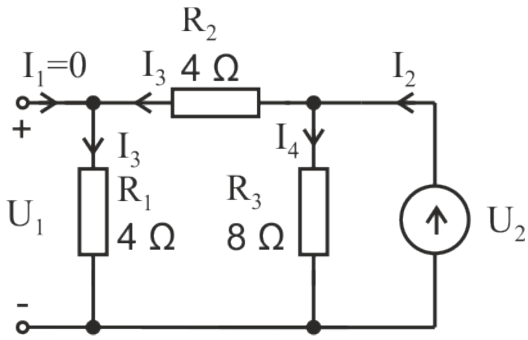
For $U_2=0$ the system becomes:

$$\begin{vmatrix}U_1=H_{11}I_1+H_{12}U_2\\ I_2=H_{21}I_1+H_{22}U_2\end{vmatrix}\rightarrow\begin{vmatrix}U_1=H_{11}I_1+0\\ I_2=H_{21}I_1+0\end{vmatrix}$$

and the other two parameters are:

$$H_{11}=\frac{U_1}{I_1}\,H_{21}=\frac{I_2}{I_1}$$

Let us first examine the two port with an open circuit at the input ($I_1=0$), and power its output with a voltage source U_2 . The equivalent circuit is:



We need to obtain U_1 and I_2 (U_2 is already known). R_1 and R_2 are connected in series, so their equivalent resistance is:

$$R_{12}=R_1+R_2=4+4=8[\Omega]$$

R_{12} is connected in parallel to the ideal source U_2 so the current I_3 would be:

$$I_3=\frac{U_2}{R_{12}}=\frac{U_2}{8}$$

Since U_1 is the voltage drop on R_1 we have:

$$U_1=R_1 I_3=4 I_3=4\frac{U_2}{8}=0{,}5 U_2$$

The equivalent resistance of all resistors is:

$$R_{123}=\frac{R_{12}.R_3}{R_{12}+R_3}=\frac{8.8}{8+8}=4[\Omega]$$

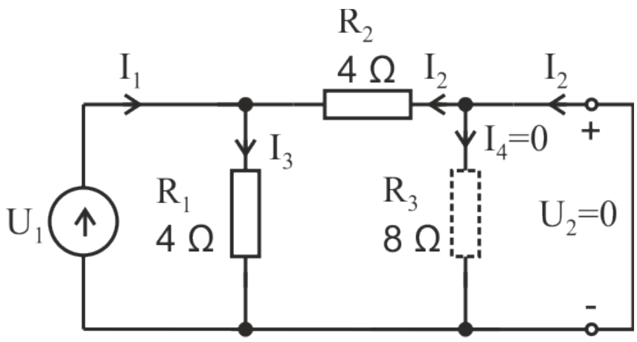
Then the current I_2 is:

$$I_2=\frac{U_2}{R_{123}}=0{,}25 U_2$$

Now we can obtain two of the hybrid parameters:

$$H_{12}=\frac{U_1}{U_2}=\frac{0{,}5 U_2}{U_2}=0{,}5 \qquad H_{22}=\frac{I_2}{U_2}=\frac{0{,}25 U_2}{U_2}=0{,}25[S]$$

Next we'll analyze the two-port with a short circuit at the output ($U_2=0$):



The voltage U_1 is known and we need to obtain I_1 and I_2 . The resistor R_3 is shunted by the short circuit, so in the circuit remain only R_1 and R_2 . We can write the following KVL equation:

$$U_1=-R_2.I_2 \quad \rightarrow \quad U_1=-4 I_2 \quad \rightarrow \quad I_2=\frac{-1}{4}U_1=-0{,}25 U_1$$

The equivalent resistance seen from the voltage source is:

$$R_{12}=\frac{R_1.R_2}{R_1+R_2}=2[\Omega]$$

According to Ohm's law the current I_1 is:

$$I_1=\frac{U_1}{R_{12}}=\frac{1}{2}U_1=0{,}5\,U_1$$

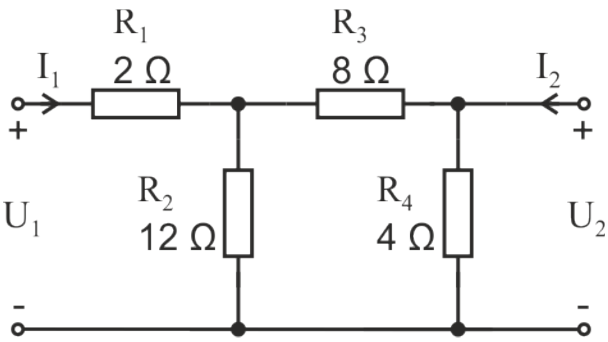
Then the other two hybrid parameters are:

$$H_{11}=\frac{U_1}{I_1}=\frac{U_1}{0{,}5\,U_1}=2[\Omega]\,H_{21}=\frac{I_2}{I_1}=\frac{-0{,}25\,U_1}{0{,}5\,U_1}=-0{,}5$$

and the H parameters of the two-port are:

$$H=\begin{bmatrix}2[\Omega] & 0{,}5\\ -0{,}5 & 0{,}25[S]\end{bmatrix}$$

Problem 2. Obtain the Z parameters of the two-port.



The system with the Z parameters is:

$$\begin{vmatrix}\dot{U}_1=Z_{11}\dot{I}_1+Z_{12}\dot{I}_2\\ \dot{U}_2=Z_{21}\dot{I}_1+Z_{22}\dot{I}_2\end{vmatrix}$$

So we need to solve the system for $I_1=0$ and for $I_2=0$.

For $I_1=0$ from the system with the Z parameters we obtain two of them:

$$U_1=Z_{11}I_1+Z_{12}I_2=0+Z_{12}I_2 \quad\rightarrow\quad Z_{12}=\frac{U_1}{I_2}$$

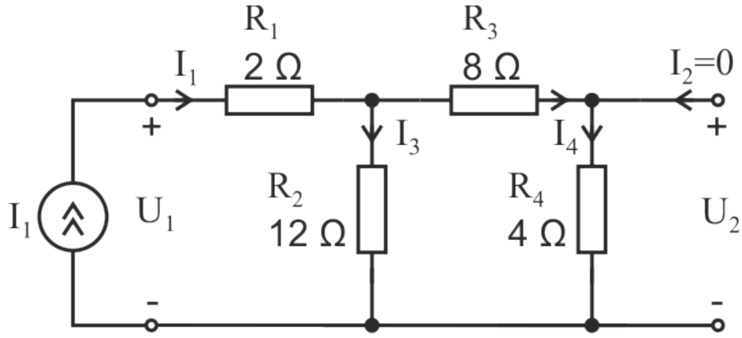
$$U_2=Z_{21}I_1+Z_{22}I_2=0+Z_{22}I_2 \quad\rightarrow\quad Z_{22}=\frac{U_2}{I_2}$$

For $I_2=0$ we obtain the other two parameters:

$$U_1=Z_{11}I_1+Z_{12}I_2=Z_{11}I_1+0 \quad\rightarrow\quad Z_{11}=\frac{U_1}{I_1}$$

$$U_2=Z_{21}I_1+Z_{22}I_2=Z_{21}I_1+0 \quad\rightarrow\quad Z_{21}=\frac{U_2}{I_1}$$

Let’s first examine the two-port with an open circuit at the output ($I_2=0$) when the input is powered by a current source I_1 :



We need to obtain U_1 and U_2. The resistors R_3 and R_4 are connected in series and then in parallel to R_2, so their equivalent resistance is:

$$R_{234}=\frac{R_2.(R_3+R_4)}{R_2+R_3+R_4}$$

The voltage drop on R_{234} is:

$$U_{234}=I_1 R_{234}=I_1\frac{R_2.(R_3+R_4)}{R_2+R_3+R_4}$$

and the currents I_3 and I_4 are:

$$I_3=\frac{U_{234}}{R_2}=I_1\frac{R_2.(R_3+R_4)}{R_2+R_3+R_4}.\frac{1}{R_2}=I_1\frac{8+4}{12+8+4}=0{,}5\,I_1$$
$$I_4=\frac{U_{234}}{R_3+R_4}=I_1\frac{R_2.(R_3+R_4)}{R_2+R_3+R_4}.\frac{1}{(R_3+R_4)}=I_1\frac{12}{12+8+4}=0{,}5\,I_1$$

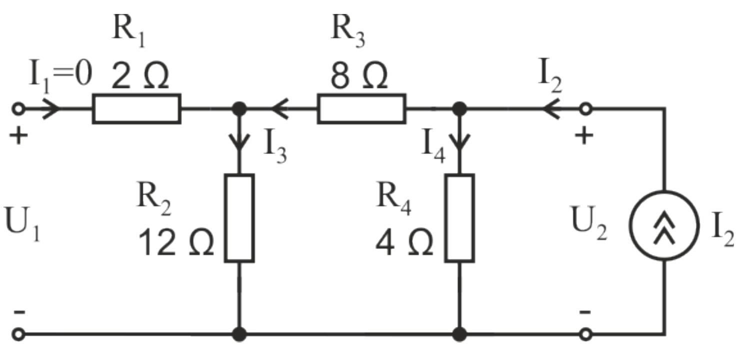
To obtain the voltages U_1 and U_2 we write two KVL equations:

$$U_1=R_1 I_1+R_2 I_3=2I_1+12I_3=2I_1+12.0{,}5\,I_1=8\,I_1$$
$$U_2=R_4 I_4=4\,I_4=4.0{,}5\,I_1=2I_1$$

Then two of the Z parameters are:

$$Z_{11}=\frac{U_1}{I_1}=\frac{8\,I_1}{I_1}=8[\Omega] \qquad Z_{21}=\frac{U_2}{I_1}=\frac{2\,I_1}{I_1}=2[\Omega]$$

Next we'll analyze the circuit for open circuit at the input ($I_1=0$) with a current source I_2 at the output:



Again we need to obtain U_1 and U_2 but this time we'll write a system of equations:

$$\begin{vmatrix} I_2=I_3+I_4 \\ 0=(8+12)I_3-4I_4 \end{vmatrix} \rightarrow \begin{bmatrix} I_3 \\ I_4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 20 & -4 \end{bmatrix}=\begin{bmatrix} I_2 \\ 0 \end{bmatrix}$$

The determinants are:

$$\Delta=\begin{bmatrix}1 & 1\\ 20 & -4\end{bmatrix}=-4-20=-24$$

$$\Delta_3=\begin{bmatrix}I_2 & 1\\ 0 & -4\end{bmatrix}=-4I_2$$

$$\Delta_4=\begin{bmatrix}1 & I_2\\ 20 & 0\end{bmatrix}=-20I_2$$

And the currents are:

$$I_3=\frac{\Delta_3}{\Delta}=\frac{-4I_2}{-24}=0{,}167I_2$$

$$I_3=\frac{\Delta_3}{\Delta}=\frac{-20I_2}{-24}=0{,}83I_2$$

To obtain U_1 and U_2 we write 2 KVL equations:

$$U_1=R_1I_1+R_2I_3=2I_1+12I_3=0+12.0{,}1667I_2=2I_2$$
$$U_2=R_4I_4=4I_4=4.0{,}8333I_2=3{,}33I_2$$

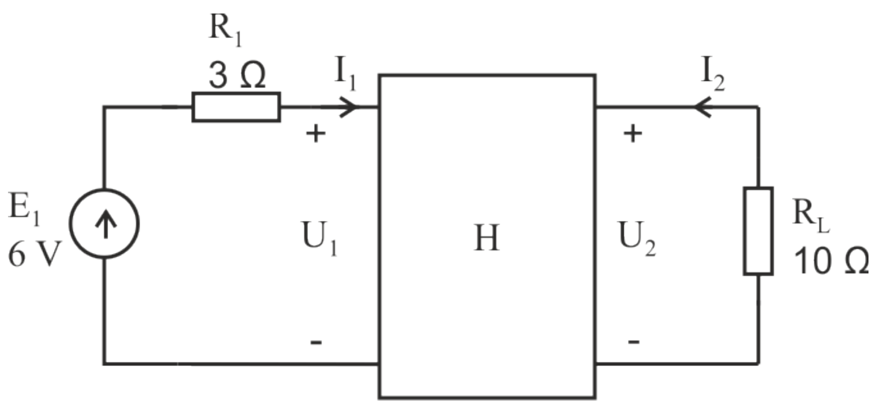
Now we can obtain the other two Z parameters:

$$Z_{12}=\frac{U_1}{I_2}=\frac{2I_2}{I_2}=2[\Omega] \qquad Z_{22}=\frac{U_2}{I_2}=\frac{3{,}33I_2}{I_2}=3{,}33[\Omega]$$

To summer, the Z parameters of the two-port are:

$$Z=\begin{vmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{vmatrix}=\begin{vmatrix}8[\Omega] & 2[\Omega]\\ 2[\Omega] & 3{,}33[\Omega]\end{vmatrix}$$

Problem 3. Obtain the current and power of the load R_L , if the two-port is defined with its hybrid parameters: $H=\begin{bmatrix}100[\Omega] & 0{,}01\\ 10 & 200[mS]\end{bmatrix}$.



First we write a system of equations using Kirchoff's laws:

$$\begin{vmatrix}6=3I_1+U_1\\ 0=U_2+10I_2\end{vmatrix}$$

We know that the system of hybrid equations is:

$$\left|\begin{array}{l}\dot{U}_1=H_{11}\dot{I}_1+H_{12}\dot{U}_2\\ \dot{I}_2=H_{21}\dot{I}_1+H_{22}\dot{U}_2\end{array}\right. \rightarrow \left|\begin{array}{l}U_1=100I_1+0{,}01\,U_2\\ I_2=10I_1+0{,}200\,U_2\end{array}\right.$$

We substitute the last equations in our system and obtain:

$$\left|\begin{array}{l}6=3I_1+U_1\\ 0=U_2+10I_2\end{array}\right. \rightarrow \left|\begin{array}{l}6=3I_1+100I_1+0{,}01U_2\\ 0=U_2+10(10I_1+0{,}2U_2)\end{array}\right. \rightarrow \left|\begin{array}{l}6=103I_1+0{,}01U_2\\ 0=3U_2+100I_1\end{array}\right.$$

In matrix form:

$$\begin{bmatrix}I_1\\U_2\end{bmatrix}\begin{bmatrix}103 & 0{,}01\\100 & 3\end{bmatrix}=\begin{bmatrix}6\\0\end{bmatrix}$$

The determinants are:

$$\Delta=\begin{bmatrix}103 & 0{,}01\\100 & 3\end{bmatrix}=103.3-100.0{,}01=308$$

$$\Delta_2=\begin{bmatrix}103 & 6\\100 & 0\end{bmatrix}=0-6.100=-600$$

Then the load voltage is:

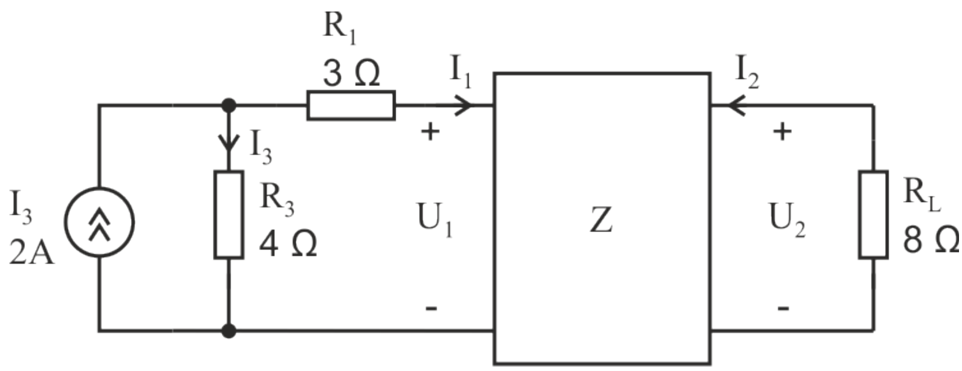
$$U_2=\frac{\Delta_2}{\Delta}=\frac{-600}{308}=-1{,}95[V]$$

Finally the current and power of the load are:

$$0=U_2+10I_2 \quad \rightarrow \quad I_2=\frac{-U_2}{R_T}=\frac{-(-1{,}95)}{10}=195[mA]$$

$$P_{RL}=U_2(-I_2)=(-1{,}95).(-195.10^{-3})=0{,}38[W]$$

Problem 4. Obtain the current and power of the load R_L, if the two-port is defined with its Z parameters: $Z=\begin{bmatrix}8[\Omega] & -3[\Omega]\\-4[\Omega] & 9[\Omega]\end{bmatrix}$.



First we are going to write a system of equations:

$$\begin{vmatrix} 2=I_1+I_3 \\ U_2=-8I_2 \\ U_1=4I_3-3I_1 \end{vmatrix}$$

The system of Z equations is:

$$\begin{vmatrix} \dot{U}_1=Z_{11}\dot{I}_1+Z_{12}\dot{I}_2 \\ \dot{U}_2=Z_{21}\dot{I}_1+Z_{22}\dot{I}_2 \end{vmatrix} \rightarrow \begin{vmatrix} U_1=8I_1-3I_2 \\ U_2=-4I_1+9I_2 \end{vmatrix}$$

Then our system of Kirchoff's laws equations becomes:

$$\begin{vmatrix} 2=I_1+I_3 \\ U_2=-8I_2 \\ U_1=4I_3-3I_1 \end{vmatrix} \rightarrow \begin{vmatrix} 2=I_1+I_3 \\ -4I_1+9I_2=-8I_2 \\ 8I_1-3I_2=4I_3-3I_1 \end{vmatrix} \rightarrow \begin{vmatrix} 2=I_1+I_3 \\ 0=4I_1-17I_2 \\ 0=3I_2+4I_3-11I_1 \end{vmatrix}$$

In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 4 & -17 & 0 \\ -11 & 3 & 4 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

The determinants are:

$$\Delta=\begin{bmatrix} 1 & 0 & 1 \\ 4 & -17 & 0 \\ -11 & 3 & 4 \end{bmatrix}=-68+12-187=-234$$

$$\Delta_2=\begin{bmatrix} 1 & 2 & 1 \\ 4 & 0 & 0 \\ -11 & 0 & 4 \end{bmatrix}=-32$$

Then the load current is:

$$I_2=\frac{\Delta_2}{\Delta}=\frac{-32}{-234}=0{,}14[A],$$

and the power dissipated by the load is:

$$P_{RL}=I_2^2R_T=0{,}14^2.8=0{,}15[W]$$