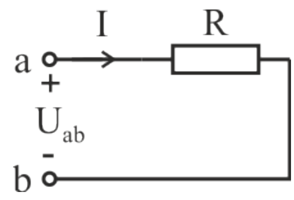
DC STEADY STATE CIRCUIT ANALYSIS

1. Introduction

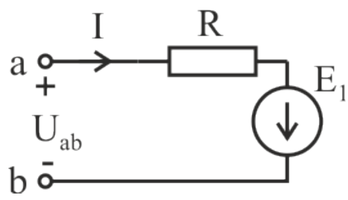
The basic quantities in electric circuits are current, voltage and resistance. They are related with Ohm's law. For a passive branch the current is:



$$I=\frac{U_{ab}}{R}=\frac{U_a-U_b}{R}$$

where U_a and U_b are the node voltages (potentials) of nodes **a** and **b**.

For an active branch, where the voltage source E_1 has the same direction as the current flow *I*, Ohm's law has the following form:



$$I=\frac{U_a-U_b+E_1}{R}=\frac{U_{ab}+E_1}{R}$$

Kirchoff's current law (KCL) refers to a node (junction) in the circuit and says**:** the sum of the currents entering the node is equal to the sum of the currents leaving it:

$$\sum I_{IN}=\sum I_{OUT}$$

Kirchoff's voltage law (KVL) refers to a loop in the circuit and says: the algebraic sum of the voltage drops in every closed loop equals zero:

$$\sum U_k=0$$

However usually it's more convenient to write it down as: the algebraic sum of the voltage sources in a closed loop is equal to the algebraic sum of the voltage drops in the loop:

$$\sum E_k=\sum U_k=\sum I_k R_k$$

Actually Ohm's law is a private case of KVL.

Power in DC electric circuits is estimated according to:

$$P=U.I$$

If in the above equation we substitute Ohm's law, the power could also be expressed as:

$$P=I^2.R \qquad P=\frac{U^2}{R}$$

For every electric circuits is in force the conservation of power: The sum of the power of the sources is equal to the sum of the power of the consumers:

$$\sum P_{SRC}=\sum P_{CONS}$$

2. Problems

Problem 1. Determine current, voltage, power and resistance of the resistor, for the following cases:

a) The current flow through the resistor $R_1=100\,\Omega$ is $I_1=10\,A$;

b) The voltage drop on the resistor $R_2=50\,\Omega$ is $U_2=25\,V$;

c) The dissipated power by the resistor $R_3=120\,\Omega$ is $P_3=1\,W$;

d) The current and dissipated power by the resistor are $I_4=1A$ and $P_4=1W$;

e) The voltage drop and dissipated power of the resistor are $U_5=12V$ and $P_5=2W$.

Solutions:

a) According to Ohm's law the voltage drop is:

$$U_1=I_1R_1=10.100=1000[V]$$

Then the dissipated power is:

$$P_1=U_1I_1=10.1000=10000=10[kW]$$

b) According to Ohm's law, the current is:

$$I_2=\frac{U_2}{R_2}=\frac{25}{50}=0,5[A]$$

Then the dissipated power is:

$$P_2=U_2I_2=25.0,5=12,5[W]$$

c) The current can be obtained from the power:

$$P_3=I_3^2R_3\rightarrow I_3^2=\frac{P_3}{R_3}\rightarrow I_3=\sqrt{\frac{P_3}{R_3}}=\sqrt{\frac{1}{120}}=0,091=91[mA]$$

Then the voltage drop is:

$$U_3=I_3R_3=0,091.120=11[V]$$

d) We can estimate the resistance from the power:

$$P_4=I_4^2R_4 \quad \rightarrow \quad R_4=\frac{P_4}{I_4^2}=\frac{1}{1^2}=1[\Omega]$$

The voltage drop is estimated according to Ohm's law:

$$U_4=I_4R_4=1.1=1[V]$$

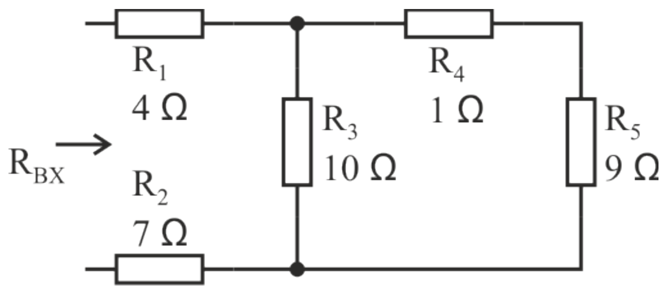
e) The current is estimated from the power law:

$$P_5=I_5U_5 \quad \rightarrow \quad I_5=\frac{P_5}{U_5}=\frac{2}{12}=0,167[A]$$

Then the resistance is:

$$R_5=\frac{U_5}{I_5}=\frac{12}{0,167}=72[\Omega]$$

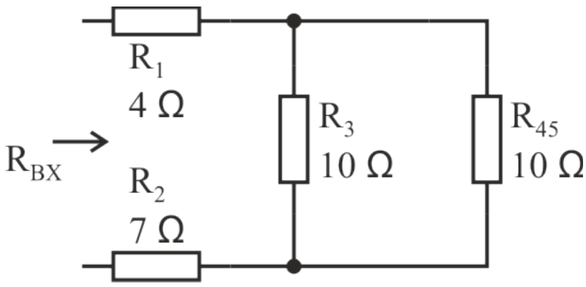
Task 2. Obtain the equivalent resistance R_{IN} and estimate what power will be dissipated in the circuit if it is powered with voltage $U=2V$.



We begin to simplify the circuit, by merging parallel and series resistors. R_4 and R_5 are connected in series, so we can combine them:

$$R_{45}=R_4+R_5=1+9=10\,\Omega$$

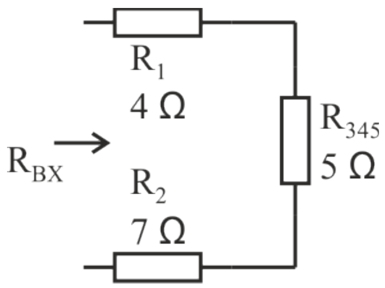
Now we can create an equivalent circuit:



Next R_3 and R_{45} are connected in parallel, so their equivalent resistance is:

$$R_{345}=\frac{R_3 R_{45}}{R_3+R_{45}}=\frac{10.10}{10+10}=5\,\Omega$$

We create a new equivalent circuit:



There we have 3 resistors in series:

$$R_{BX}=R_1+R_2+R_{345}=4+7+5=16\,\Omega$$

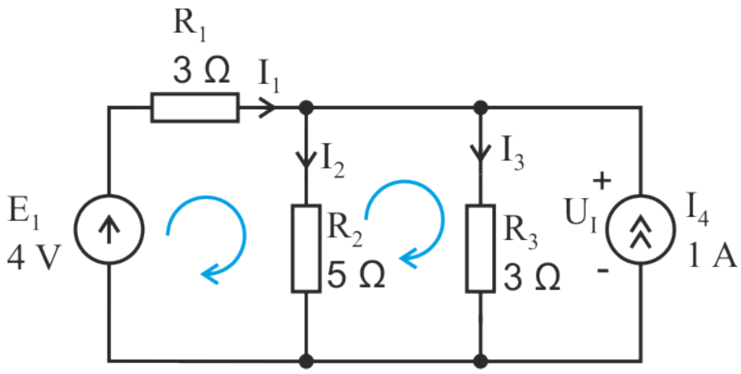
If we power the circuit with voltage $U=2V$, the input current is:

$$I=\frac{U}{R_{BX}}=\frac{2}{16}=0{,}125[A]$$

Then the power, dissipated in the circuit, is:

$$P=U.I=2.0{,}125=0{,}25[W]$$

Task 3. Obtain the branch currents in the circuit using the Kirchoff's laws and verify the conservation of power.



The circuit has three unknown currents (the 4th one is known because there is a current source), so we need a system of three equations. The circuit has two nodes, so we can write 1 equation using KCL and 2 equation using KVL:

$$\left| \begin{array}{l} I_1+I_4=I_2+I_3 \\ E_1=R_1.I_1+R_2.I_2 \\ 0=-R_2.I_2+R_3.I_3 \end{array} \right. \rightarrow \left| \begin{array}{l} I_1+1=I_2+I_3 \\ 4=3.I_1+5.I_2 \\ 0=-5.I_2+3.I_3 \end{array} \right.$$

We write the system of equations in matrix form:

$$\begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} \begin{vmatrix} -1 & 1 & 1 \\ 3 & 5 & 0 \\ 0 & -5 & 3 \end{vmatrix} = \begin{vmatrix} 1 \\ 4 \\ 0 \end{vmatrix}$$

The determinants are:

$$\Delta = \begin{vmatrix} -1 & 1 & 1 \\ 3 & 5 & 0 \\ 0 & -5 & 3 \end{vmatrix} = -1.5.3-1.5.3-1.3.3=-39$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 0 \\ 0 & -5 & 3 \end{vmatrix} = 1.5.3-1.4.5-1.3.4=-17$$

$$\Delta_2 = \begin{vmatrix} -1 & 1 & 1 \\ 3 & 4 & 0 \\ 0 & 0 & 3 \end{vmatrix} = -1.4.3-1.3.3=-21$$

$$\Delta_3 = \begin{vmatrix} -1 & 1 & 1 \\ 3 & 5 & 4 \\ 0 & -5 & 0 \end{vmatrix} = -1.3.5-1.4.5=-35$$

Then the currents are:

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-17}{-39} = 0{,}436\,[A]$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-21}{-39} = 0{,}539\,[A]$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-35}{-39} = 0{,}897\,[A]$$

Conservation of power

In order to verify the conservation of power we need the voltage drop on the current source. We could obtain it from a loop though the current source and R_3. The KVL is:

$$U_I - R_3.I_3 = 0 \quad \rightarrow \quad U_I = R_3.I_3 = 3.I_3 = 3.0{,}897 = 2{,}691\,[V]$$

Then the power of the sources is:

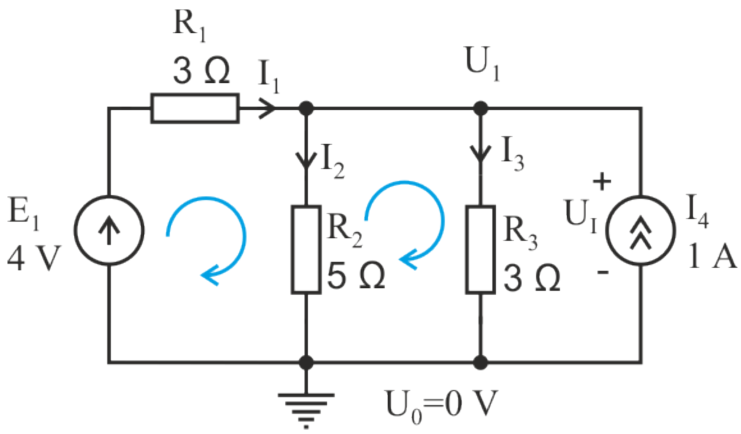
$$P_{SRC} = P_{E1} + P_{I4} = E_1.I_1 + U_I.I_4 = 4.0{,}436 + 2{,}691 = 4{,}44\,[W]$$

The power of the consumers is:

$$P_{CONS} = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 = 0{,}436^2.3 + 0{,}539^2.5 + 0{,}897^2.3 = 4{,}44\,[W]$$

Since $P_{SRC} = P_{CONS}$ the power is conserved, which means we have solved the circuit correctly.

Task 4. Obtain the currents in the circuit using nodal analysis.



The circuit has 2 nodes. We connect the lower one (n. 0) to ground so it has node voltage $U_0 = 0V$. Then the unknown node voltage is U_1.

We can write a KCL equation for node 1:

$$I_1 + 1 = I_2 + I_3$$

The three currents can be estimated using Ohm's law:

$$I_1 = \frac{U_0 - U_1 + 4}{3} = \frac{-U_1 + 4}{3}$$

$$I_2 = \frac{U_1 - U_0}{5} = \frac{U_1}{5}$$

$$I_3 = \frac{U_1 - U_0}{3} = \frac{U_1}{3}$$

We substitute the three currents in the KCL equation:

$$I_1 + 1 = I_2 + I_3 \quad \rightarrow \quad \frac{-U_1 + 4}{3} + 1 = \frac{U_1}{5} + \frac{U_1}{3} \quad \rightarrow \quad U_1\left(\frac{1}{3} + \frac{1}{5} + \frac{1}{3}\right) = \left(1 + \frac{4}{3}\right)$$

Then the node voltage is:

$$U_1 = 2{,}692\,[V]$$

Now we can find the three currents:

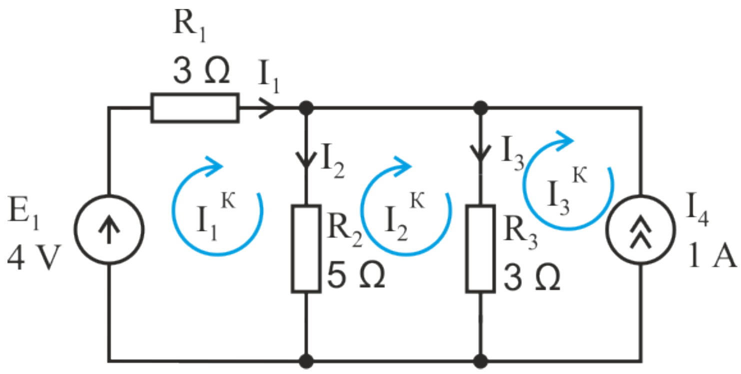
$$I_1 = \frac{-U_1 + 4}{3} = \frac{-2{,}692 + 4}{3} = 0{,}436[A]$$

$$I_2 = \frac{U_1}{5} = \frac{2{,}692}{5} = 0{,}539[A]$$

$$I_3 = \frac{U_1}{3} = \frac{2{,}692}{3} = 0{,}897[A]$$

Note that we obtained results using Kirchoff's law method, but this time there was only 1 equation and 1 unknown.

Task 5. Obtain the currents in the circuit using Mesh analysis.



We define enough number of loops, so that all circuit elements are included and all loops rotate in the same direction (clockwise). One of the mesh currents (I_3^K) goes through the current source with an opposite direction, so its size is known:

$$I_3^K = -I_4 = -1[A]$$

For the other two loops we write KVL equations:

$$E_1 = I_1^K(R_1 + R_2) - I_2^K R_2 \quad \rightarrow \quad 8I_1^K - 5I_2^K = 4$$

$$0 = I_2^K(R_2 + R_3) - I_1^K R_2 - I_3^K R_3 \quad \rightarrow \quad 8I_2^K - 5I_1^K = -3$$

We write the system of equations in matrix form:

$$\begin{vmatrix} I_1^K \\ I_2^K \end{vmatrix} \begin{vmatrix} 8 & -5 \\ -5 & 8 \end{vmatrix} = \begin{vmatrix} 4 \\ -3 \end{vmatrix}$$

The determinants are:

$$\Delta = \begin{vmatrix} 8 & -5 \\ -5 & 8 \end{vmatrix} = 64 - 25 = 39$$

$$\Delta_1 = \begin{vmatrix} 4 & -5 \\ -3 & 8 \end{vmatrix} = 32 - 15 = 17$$

$$\Delta_2 = \begin{vmatrix} 8 & 4 \\ -5 & -3 \end{vmatrix} = -24 + 20 = -4$$

Then the two mesh currents are:

$$I_1^K = \frac{\Delta_1}{\Delta} = \frac{17}{39} = 0{,}436[A]$$

$$I_2^K = \frac{\Delta_2}{\Delta} = \frac{-4}{39} = -0{,}103[A]$$

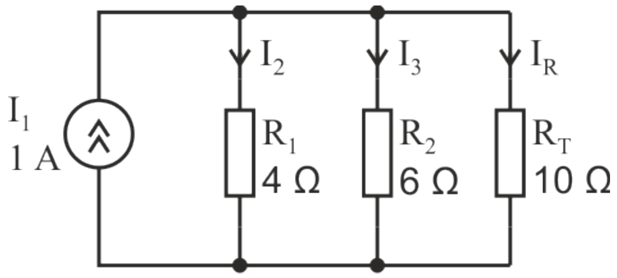
The branch currents are an algebraic sum of the mesh currents, going through them:

$$I_1 = I_1^K = 0{,}436[A]$$

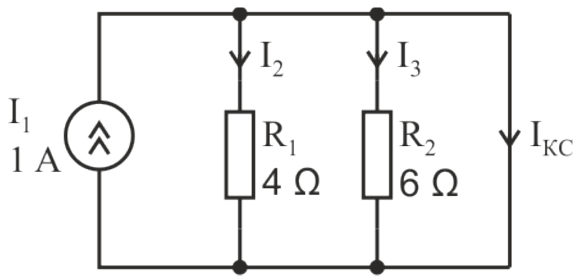
$$I_2 = I_1^K - I_2^K = 0{,}436 + 0{,}103 = 0{,}539[A]$$

$$I_3 = I_2^K + 1 = 0{,}897[A]$$

Task 6. Analyze the circuit using Thevenin's theorem and obtain the power dissipated by the load R_T .



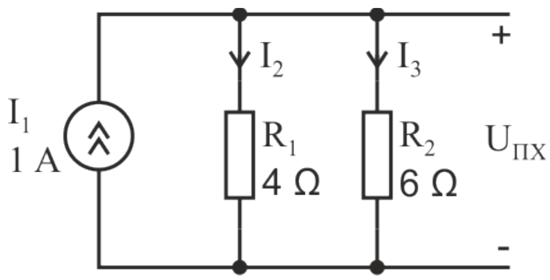
First we obtain the short circuit current of the equivalent Thevenin generator:



The resistors R_1 and R_2 are shunted, so there will be no voltage drop on them. Then the short circuit current is:

$$I_{KC} = I_1 = 1[A]$$

Now we need to obtain the open circuit voltage $U_{ПХ}$



The equivalent resistance of R_1 and R_2 is:

$$R_{EKB} = \frac{R_1 R_2}{R_1 + R_2} = \frac{4.6}{4+6} = 2{,}4[\Omega]$$

Then the open circuit voltage can be obtained using Ohm's law:

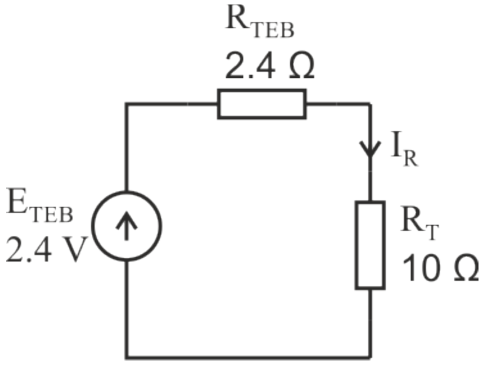
$$U_{ПХ} = I_1 R_{ЕКВ} = 2{,}4.1 = 2{,}4 [V]$$

So the Thevening voltage source has the following characteristics:

$$E_{ТЕВ} = U_{ПХ} = 2{,}4 [V]$$

$$R_{ТЕВ} = \frac{U_{ПХ}}{I_{КС}} = \frac{2{,}4}{1} = 2{,}4 [\Omega]$$

and the Thevenin equivalent circuit is:



Then the load's current and the dissipated power are:

$$I_R = \frac{E_{ТЕВ}}{R_{ТЕВ} + R_T} = \frac{2{,}4}{2{,}4 + 10} = 0{,}194 [A]$$

$$P_{RT} = I_R^2 R_T = 0{,}194^2.10 = 0{,}375 [W]$$